# PREDITION OF DEFLECTION IN REINFORCED CONCRETE BEAMS CONTAINING PLASTIC WASTE

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**Abstract**
Serviceability limit state is one of the required criteria that must be met in any analysis and design procedure. All concrete design codes have focused on the calculation methods for these criteria. One of these criteria is the deflection, which depend on the flexural stiffness (EI) of the studied element. Two methods for determining the effective flexural stiffness were studied: Branson's method that was derived in 1965, and Bischoff's method that was derived in 2005. Then these methods were used to verify an experimental work conducted for reinforced concrete beams with light weight aggregates. It has been found that the load deflection behavior of reinforced concrete beams with light weight aggregates were different from that with normal weight. A theoretical equation for each beam with lightweight aggregates was derived and compared with the real experimental curve. Finally, conclusions and recommendations were stated.

**Keywords:**
Deflection prediction, Flexural, Prediction, Reinforced concrete beam, Waste Plastic.

## 1 INTRODUCTION

Any structural design procedure needs to meet two requirements: The ultimate limit state requirements which is related to sections strength and capacity, and the serviceability limit state that is related to the workability of the section. Several examples of serviceability limit state components are: Deflection, Crack width, and vibration. Deflection is a very important component that must satisfy code limits specially for structures that contain liquids such as water or oil. The main parameters that effects the deflection id the bending stiffness of the element that is related to main components: the moment of inertia (I) and the modules of elasticity (E).

Equation 1 shows the ACI 318 [ACI 2005] equation for determining the modulus of elasticity:

$$E_c = {w_c}^{1.5} \times 0.043 \times \sqrt{f'c} \qquad (1)$$

$$E_c = 4700 \times \sqrt{f'c} \qquad (2)$$

Where "$E_c$" is the modulus of elasticity, "$w_c$" is concrete density, and "$f'_c$" is the concrete compressive strength. Equation 1 is used for light weight concrete whereas equation 2 is used for normal weight concrete.

The post elastic behavior of concrete results in the variation in the modulus of elasticity. The second moment of area is a function of the tensile strength capacity of the concrete. The concrete becomes ineffective once the tensile strength is reached and the tensile stresses will be predominantly resisted by the steel. Therefore, the second moment of area will vary depending upon the moment where the first crack appeared. The uncracked second moment of area is given by Equation 3 as follows:

$$I_{ucr} = \frac{1}{12}bh^3 + bh\left(y' - \frac{h}{2}\right)^2 + (n-1)A_s(d-y')^2 \qquad (3)$$

Where equation 3 parameters are illustrated in Fig. 1.

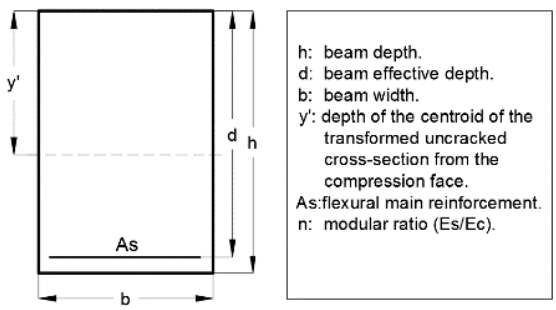


*Fig. 1 Transformed area parameters for uncracked beam.*

An approximated equation is used for the uncracked moment of inertia. This method neglects the effect of steel reinforcement, hence equation 3 will become:

$$I_g = \frac{1}{12}bh^3 \qquad (4)$$

As the applied load increases, cracks in the beam will appear and the moment is referred to as the cracking moment ($M_{cr}$). Cracks will propagate upwards till they reach the neutral axis. At this stage the section will become fully cracked and the second moment of area will vary and can be calculated using equation (5) as follows:

$$I_{cr} = \frac{1}{12}bc^3 + nA_s(d-c)^2 \quad (5)$$

Where equation 3 parameters are illustrated in Fig. 2.

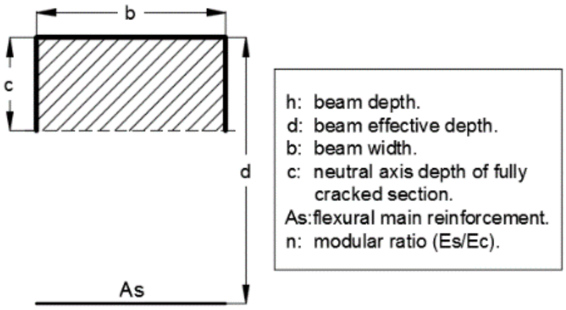


*Fig. 2 Transformed area parameters for cracked beam.*

As the applied moment increases the value of cracked moment ($M_{cr}$), the moment of inertia will decrease gradually from ($I_{ucr}$) until reaching ($I_{cr}$) when the section id fully cracked. [Branson 1965] suggested an equation in 1965 that represent the value of the effective moment of inertia ($I_{eff}$) as the following:

$$I_{eff} = \left(\frac{M_{cr}}{M_a}\right)^3 . I_{unc} + \left(1 - \left(\frac{M_{cr}}{M_a}\right)^3\right) . I_{cr} \quad (6)$$

This equation is adopted by ACI 318-05 [ACI 2005] , AASHTO LRFD [ASSHTO 2005], CSA A23.3-04 [CSA 2004], AS 3600 [SAA 1994] and TS 500 [TS 2000] in the immediate deflection calculations of concrete beams.

[Bischoff (2005)] and [Bischoff (2007)] suggested an equation to determine the effective second moment of area. The equation (Equation 7) was based a weighted average of the flexibilities of the uncracked and cracked portions of a reinforced concrete beam:

$$\frac{1}{I_{eff}} = \left[\left(\frac{M_{cr}}{M_a}\right)^2 . \frac{1}{I_{unc}}\right] + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^2\right] . \frac{1}{I_{cr}} \quad (7)$$

As for ($M_{cr}$) it is calculated from equation 8:

$$M_{cr} = \frac{f_r . I_g}{y_t} \quad (8)$$

where “$y_t$” is the vertical distance of the extreme tension fibers from the neutral axis.

The modulus of rupture ($f_r$) can be calculated using Equation (9) suggested by the ACI 318-05 [ACI 2005]:

$$f_r = 0.62\sqrt{f'c} \ (MPa) \quad (9)$$

The purpose of this research is to compare the load deflection behavior of light weight reinforced concrete beams with the suggested equations by Branson Bischoff. Also a newly developed equation when waste plastic is suggested.

Experimental data will be taken from [Khatib et al 2019] and will be plotted against both Branson and Bischoff so it can be evaluated. In this paper we are dealing with static loading as the behaviour under dynamic blast loading would be different [Temsah et al 2018a; Temsah et al 2018b]. Also the use of other waste materials to either replace cement or aggregate in concrete will be the subject of future investigations [Baalbaki et al. 2018; Charbaji et al 2018; El-Darwish *et al., 1997;* El-Kurdi *et al*., 2014; Herki and Khatib, 2013; Herki and Khatib, 2016; Khatib *et al*., 2008; Khatib *et al*., 2009; Khatib *et al.* 2013a; Khatib *et al.* 2013b; Khatib *et al.* 2014; Khatib, 2016 ; Khatib *et al*., 2016; Mangat *et al*., 2006; Okeyinka *et al*., 2015; Sonebi *et al*., 2016; Wright and Khatib, 2016].

Four concrete mixes were employed to study the flexural characteristics behavior of reinforced concrete beam incorporating plastic waste bottle caps [Khatib et al 2019]. The reference mix (PBC 0) had a proportion (by weight) of 1 : 3 : 3 (cement : fine aggregate : coarse aggregate and no waste plastic was used. In mixes PCB10, PCB15 and PBC 20, the coarse aggregate was partially replaced (by volume) with 10%, 15% and 20% waste plastic bottle caps (PBC) respectively. The caps have 25mm (dia) and 12mm (depth) with a thickness of about 1.5mm. All concrete mixes had the same free water to cement (W/C) ratio of 0.6. The selection of theses mixes was based on a series of trial mixes. Tab. 1 presents the details of the four concrete mixes.

*Tab. 1 Details of concrete mixes*

| | | Quantity (Kg/m$^3$) | | | | |
|---|---|---|---|---|---|---|
| Mix | R* % | Cement | Water | Sand | Gravel | PBC** |
| PBC 0 | 0 | 314 | 189 | 943 | 943 | 0.0 |
| PBC10 | 10 | 314 | 189 | 943 | 848 | 31 |
| PBC15 | 15 | 314 | 189 | 943 | 801 | 46 |
| PBC20 | 20 | 314 | 189 | 943 | 745 | 61 |

*% replacement by volume of coarse aggregate with PBC

**Plastic bottle caps

The dimensions of the Reinforced concrete beams were 200mm x 300mm x 1200mm. The main (bottom) reinforcement of all beams was 3 bars mild steel and the top reinforced had two bars. The diameter of the bottom and top reinforcement was 8 mm. Only the content of PBC varied as indicated in Tab. 1. The stirrups had a diameter of 6mm and spaced at 200mm centres. Fig. 3 shows the cross sectional area of the beam and the reinforcing bars.

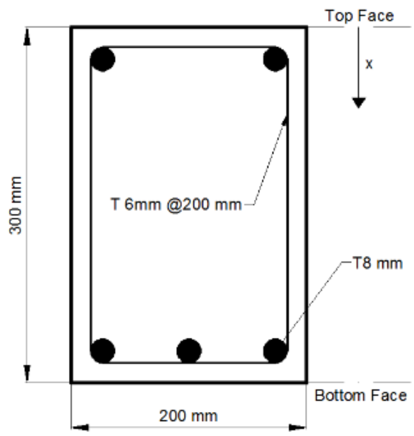


*Fig. 3 Cross section of the beam and reinforcement.*

The flexural behavior of reinforced concrete beams was experimentally assessed using the four-point test as described in a previous investigation [Khatib et al. 2015] . Fig. 4 shows the location of the supports and point loads. The load was increased in 4 kN increment and the load at first crack was noted. Then, the loading continued until failure. The mid-span deflection was recorded at each load increment. The initiation and propagation of cracks were observed throughout the test. Fig. 5 shows the load deflection curve for all concrete beams [Khatib et al 2019].

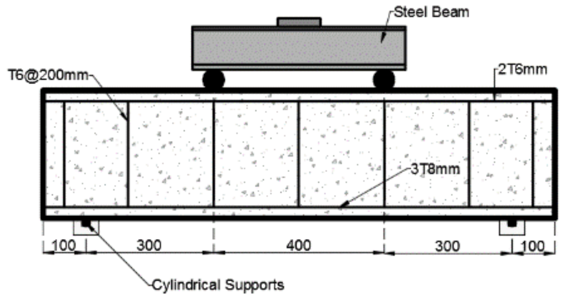


*Fig. 4 Longitudinal section of beam with supports and loads (dimensions in mm).*

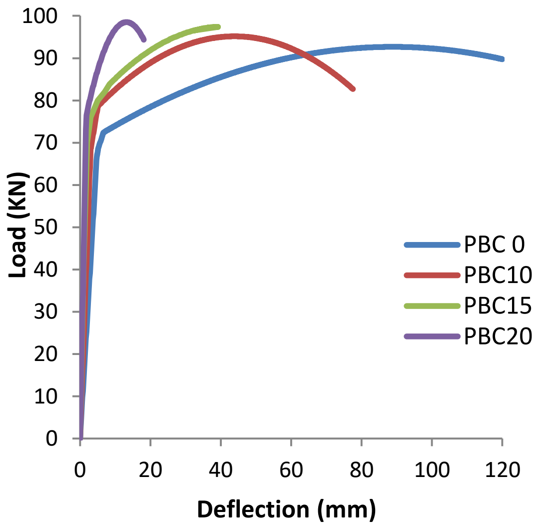


*Fig. 5 Load-Deflection Curve for all samples* [Khatib et al 2019].

## 2 APPLICATION OF BRANSON AND BISCHOFF METHODS TO THE EXPERIMENTAL DATA

Fig. 6 to 9 shows the load deflection curve for all experimental samples compared to the equation suggested by of [Branson 1965] and [Bischoff 2005]. For the beam without plastic caps (PBC0), Both Branson and Bischoff curves match the experimental curve. Whereas for the other beams that contain plastic bottle caps, there was some difference between both Branson and Bischoff curves, and the experimental curve. It can be realized that the beams with plastic caps (PBC10, PBC15 and PBC20) have a softer behavior compared to Branson and Bischoff.

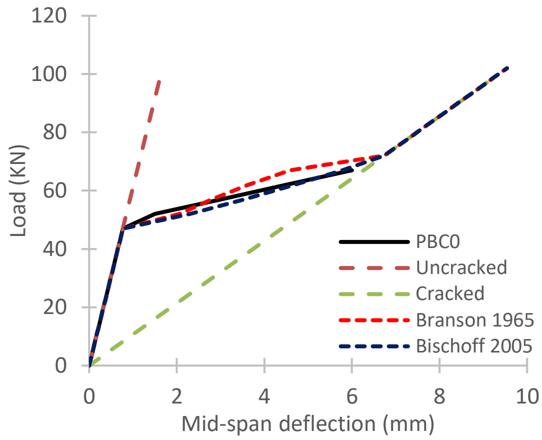


*Fig. 6 Application of [Branson 1965] and [Bischoff 2005] for PBC0.*

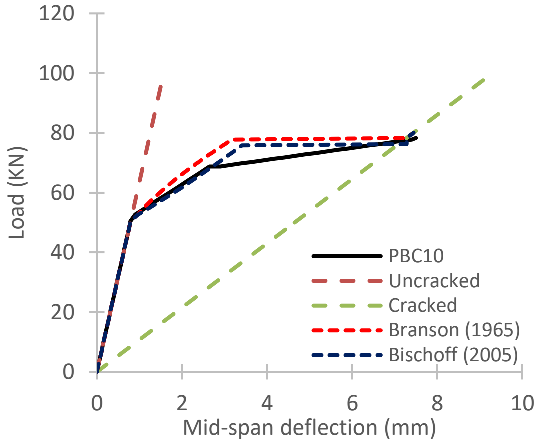


*Fig. 7 Application of [Branson 1965] and [Bischoff 2005] for PBC10.*

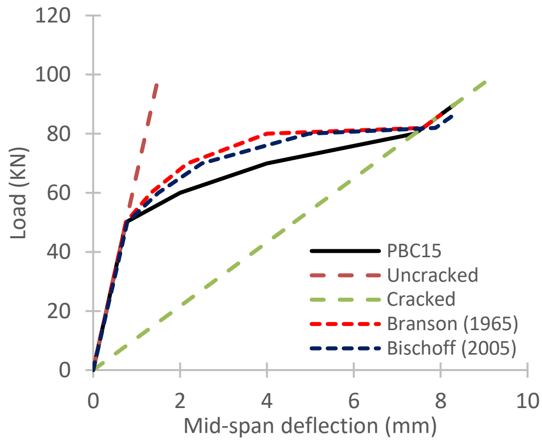


*Fig. 8 Application of [Branson 1965] and [Bischoff 2005] for PBC15*

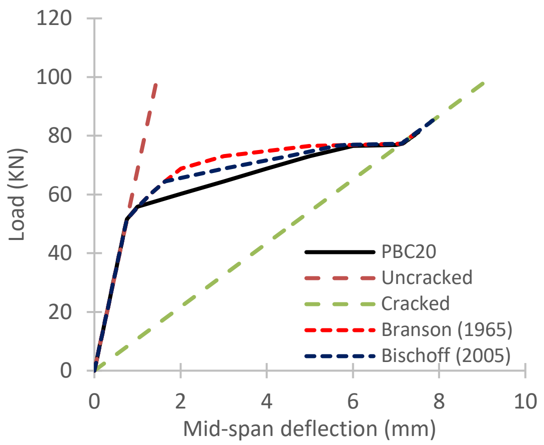


*Fig. 9 Application of [Branson 1965] and [Bischoff 2005] for PBC20*

A modification to Branson's formula that is adopted by the ACI code was used in the prediction. The power value of the term ($M_{cr}/M_a$) was determined and the effective moment of inertia was replotted based on the average power value of the term ($M_{cr}/M_a$). The average power value was 3.9 for PBC10, 5 for PBC15, and 4.2 for PBC20. Therefore, Branson's equation becomes:

For PBC10

$$I_{eff} = \left(\frac{M_{cr}}{M_a}\right)^{3.9} . I_{unc} + \left(1 - \left(\frac{M_{cr}}{M_a}\right)^{3.9}\right) . I_{cr} \tag{10}$$

For PBC15

$$I_{eff} = \left(\frac{M_{cr}}{M_a}\right)^{5} . I_{unc} + \left(1 - \left(\frac{M_{cr}}{M_a}\right)^{5}\right) . I_{cr} \tag{11}$$

For PBC20

$$I_{eff} = \left(\frac{M_{cr}}{M_a}\right)^{4.2} . I_{unc} + \left(1 - \left(\frac{M_{cr}}{M_a}\right)^{4.2}\right) . I_{cr} \tag{12}$$

Fig. 10 to 12 show the original experimental curve versus the theoretically derived curve for each beam sample.

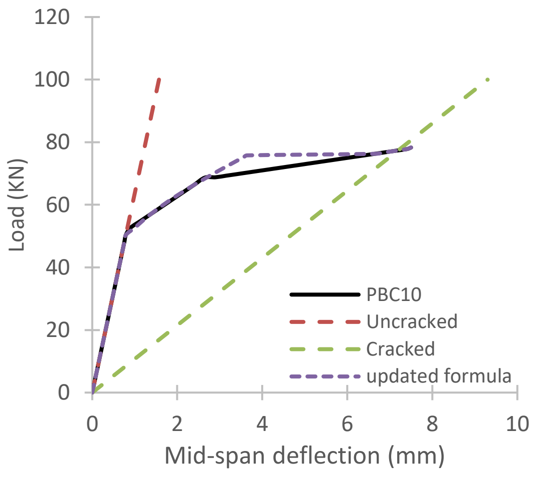


*Fig. 10 Experimental vs updated formula load deflection curve for PBC10*

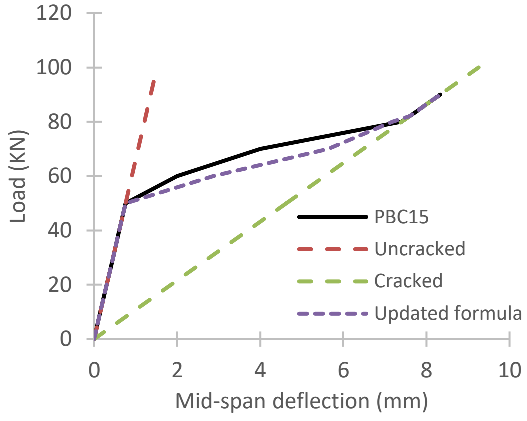


*Fig. 11 Experimental vs updated formula load deflection curve for PBC15*

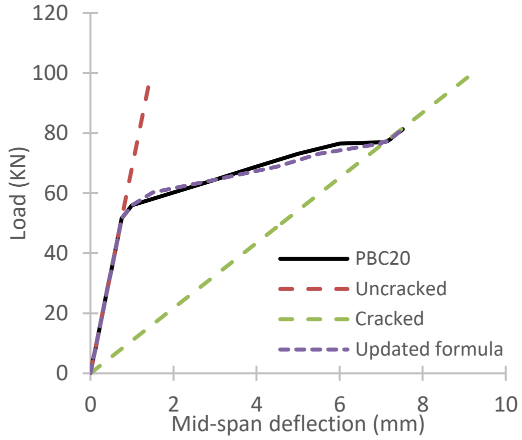


*Fig. 12 Experimental vs updated formula load deflection curve for PBC20*

## 3 CONCLUSION

The following conclusions can be made from this study:

1. The methods proposed by Branson (1965) and Bischoff (2005) closely estimate the load deflection behavior of reinforced concrete beams.

2. The method proposed by Bischoff (2005) provides a slightly better correlation with the actual load deflection curves of reinforced concrete beams.

3. The derived formula for the beams that contain plastic caps were different from that derived by Branson. The power used in these equations were all greater than the value used by Branson which is 3.

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