OPTIMIZING THE DESIGN OF TIMBER COMPONENTS UNDER DECAY AND CLIMATE VARIATIONS

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Abstract

The durability of timber structures can be affected by the isolated or combined actions of loading, moisture content, temperature, biological activity, etc. This work focuses on the optimal design of new timber structures subjected to deterioration. Since the deterioration processes and the structural behavior of timber structures are complex, nowadays the deterioration models are not able to account for all influencing factors. Consequently, this study is based on an empirical model that was derived based in-lab experimental studies for the decay growth of brown rot in pine sapwood under variant climate conditions. Such a model is divided into two processes: (i) activation process and (ii) mass loss process. On the other hand, there are significant uncertainties involved in the problem. The uncertainties inherent to materials properties, models and climate are considered throughout a time-dependent reliability based-design optimization (TD-RBDO) approach. The TD-RBDO aims to ensure a target reliability level during the operational life. This approach is applied to design optimization of a new timber component subjected to different French climates. The performance of the optimized solution is compared with a traditional cross-section designed according to the Eurocode 5 in terms of safety. The overall results indicate that an optimized solution ensures a target reliability level during the whole structural lifetime.

1 INTRODUCTION

The mechanical and physical properties of timber structures could be affected by a combination of loading, moisture content, temperature, biological activity, etc. This paper focuses on the optimal design of new timber structures subjected to fungal decay. Structural optimization is widely used for searching the optimal design cost of civil engineering structures. The Deterministic Design Optimization (DDO) procedure is successfully applied for designing concrete and steel structures [Kravanja 2013; Tomás 2010]. Other works have used the DDO approach to optimize the design of timber trusses [Šilih 2005] or finger-joints under bending solicitations [Tran 2014].

The DDO procedure is based on minimizing objective functions defined in terms of the structural volume or costs for structures and subjected to geometric, stress and deflection constraints. These design conditions are considered in accordance with Eurocode 5 [NF-EN 1995 2005) in order to satisfy the requirements of both the ultimate and the serviceability limit states. In Fact, the real benefit of the DDO approach is cost reduction and effective use of structural capacity.

The partial safety factors introduced in the deterministic design are assumed to take account for uncertainties related to timber material, structural dimension and loading. These safety factors are applied in the design constraints to ensure the safety margin. These factors are calibrated for a large class of structures. Moreover, the safety margin produced with these partial safety factors in not directly linked to uncertainties. Thus, the use of these partial safety factors in the deterministic design optimization can lead to over or under designing structures.

The Reliability-Based Design Optimization (RBDO) offers a suitable framework for the consideration of the uncertainties in the design optimization and to find the best compromise between cost reduction and safety assurance. A practical formulation of the RBDO consists of minimizing the expected cost under probabilistic constraints.

Furthermore, the long-term durability of timber structures depends on the effect of moisture that in combination with propitious temperature conditions and exposure time may deteriorate the material resistance capacity. The exposure of unprotected

timber structures to humid environments increases the moisture content inside wood leading to fungal decay. This deterioration mechanism reduces the strength capacity of timber structures affecting its serviceability and safety.

Within this framework, this paper applies a Time-dependent Reliability-Based Design Optimization approach (TD-RBDO) [Aoues 2009] to find the best design configuration of a roof structure under fungal decay. The TD-RBDO approach aims searching the optimal design that minimizes the structural cost and to ensure a target reliability level during the operational life. This approach is applied to design optimization of a timber truss subjected to a very humid French climate (Nantes City). The performance of the optimized solution is compared with the optimal design estimated by the DDO approach and the RBDO approaches. The overall results show that an optimized solution obtained by the TD-RBDO method ensure the target reliability level during the whole structural lifetime.

2 DETERMINISTIC DESIGN OPTIMIZATION

The Deterministic Design Optimization aims at minimizing an objective function as the structural volume or cost subjected to geometric, stress and deflection constraints, as defined in the design codes, for instance by the Eurocode 5:

$$\min_{\mathbf{d}=\{\mathbf{h},\mathbf{b}\}} \sum_{i=1}^{n_b} \rho L_i h_i b_i$$

$$\text{subject to} \begin{cases} G_{ULS,i}(\mathbf{d},\mathbf{x}_k,\gamma) \leq 0 \\ G_{SLS,i}(\mathbf{d},\mathbf{x}_k,\gamma) \leq 0 \\ \psi_i(\mathbf{d}) \leq 0 \end{cases} \tag{5}$$

where, $\mathbf{d}$ is the vector of design variables, for rectangular cross-section $\mathbf{d}$ is composed by the depth $\mathbf{b}$ and the breadth $\mathbf{h}$ of the members, L_i is the length of the ith member and ρ is the timber density. $G_{ULS,i}$ and $G_{SLS,i}$ are respectively the ith Ultimate Limit State (ULS) and Service Limit State (SLS). The limit state functions are defined in terms of the design variables $\mathbf{d}$, the characteristic values of load actions and material properties collected in the vector $\mathbf{x}_k$ and the partial safety factors γ and ψ are the feasibility constraints (e.g. upper and lower bounds of design variables).

3 TIME-DEPENDENT RELIABILITY BASED-DESIGN OPTIMIZATION

For structural systems, The Reliability Based Design Optimization is formulated as the minimization of the cost function under reliability constraints.

$$\min_{\mathbf{d}=\{\mathbf{h},\mathbf{b}\}} \sum_{i=1}^{n_b} \rho L_i h_i b_i$$

$$\text{subject to} \begin{cases} \beta_i(\mathbf{d},\mathbf{X}) \geq \beta_i^t \\ \psi_i(\mathbf{d}) \leq 0 \end{cases} \tag{6}$$

where $\beta_i(\mathbf{d}, \mathbf{X})$ are the reliability indexes for the ith ultimate or serviceability limit state G_i and β_i^t is the target reliability index for the ith limit state. In the above formulation, the reliability constraints define the feasible domain, such as the reliability indexes $\boldsymbol{\beta}$ are kept upper than the target reliability indexes $\boldsymbol{\beta}^t$. Several approaches have been recently developed to solve the above formulation in Eq. 2. [Aoues 2010] have compared and discussed different RBDO approaches regarding robustness and numerical performance. The SORA method [Du 2004] appears more robust and more accurate than the other methods. This approach is used in the study to search the best reliable design of the timber structures.

However, when degradation is considered, the time-dependent Reliability-Based Design optimization is proposed to find the optimal design by satisfying appropriate safety levels during the whole structure lifetime. The TD-RBDO is formulated as:

$$\min_{\mathbf{d}=\{\mathbf{h},\mathbf{b}\}} \sum_{i=1}^{n_b} \rho L_i h_i b_i$$

$$\text{subject to} \begin{cases} \beta_i(\mathbf{d},\mathbf{X},t) \geq \beta_i^{t,T_L} \quad \forall t \in [0,T_L] \\ \psi_i(\mathbf{d}) \leq 0 \end{cases} \tag{7}$$

where, $\beta_i(\mathbf{d}, \mathbf{X}, t)$ is the reliability index at the time t taken the lifetime interval $[0, T_L]$, β_i^{t,T_L} is the target reliability index at the allowable life time T_L, depending on the target reliability related to one year reference period by the following relation [NF-EN 1990 2003]:

$$\beta_{T_L}^c = \Phi^{-1}\left[\Phi\left(\beta_1^t\right)^{T_L}\right] \tag{8}$$

where β_1^t is the target reliability index for one year. The classical TD-RBDO formulation given in Eq. (3) is based on the time-dependent reliability, that aims at computing the probability of failure during the whole structure lifetime. The time dependency lies mainly in the degradation phenomena. This formulation is not suitable for real engineering structures because considerable computational effort is required, and convergence can hardly be achieved. The major drawback lies in the time-dependent reliability analysis, which requires considerable computational efforts. In this work, the Sequential Optimization and Time-Variant Reliability Analysis (SOTVRA) approach developed and implemented by [Aoues 2009] is used to perform the TD-RBDO methodology. The SOTVRA approach is based on transforming the TV-RBDO problem into a sequence of equivalent deterministic design optimization sub-problems. This transformation is defined by the mean of optimal safety factors, linking the reliability requirement to the equivalent deterministic optimization. At the end of each optimization sub-problem, the reliability constraint is verified by performing the time-dependent reliability. The safety factors corresponding to the target reliability level at the initial time are calibrated by a probabilistic approach. Finally, these safety factors are provided to the following sub-problem of the

equivalent deterministic optimization and so on, until convergence.

4 MODELING TIMBER DECAY

4.1 Background

Decay models are based on in-field or in-lab measures and report results for specific wood species and locations [Brischke 2014]. Among the models based on in-field observations, the Timber Life empirical model was derived based on 20-year data obtained by exposing 4000 small clear timber specimens in 11 sites around Australia, augmented by more than 1500 bits of data obtained from examining existing timber constructions [Leicester 2009]. This study focused on Australian species. This model has been adapted and used for many researchers worldwide [de Freitas 2010; Lourenço 2012; Ryan 2014; Sousa 2014]. However, it does not provide direct relationships between decay and specific climate conditions for a given zone that we would like to consider in an optimal design.

Isaksson et al [Isaksson 2012] also proposed a decay model based on in field measurements on Scots pine sapwood (*Pinus sylvestris L.*) and Douglas fir heartwood (*Pseudotsuga menziesii Franco*) specimens. The specimens were subjected to real exposure conditions in 24 European test sites with different climate regimes between 2000 and 2008. The model developed by [Isaksson 2012] aims at estimating decay in terms of 'dose-response functions'. The 'dose' is expressed as a function of daily wood moisture content and wood temperature and the level of decay is defined according to [EN 252 1990]. [EN 252 1990) proposes 5 levels of decay condition from no decay (sound) until very severe decay. These levels are useful when condition assessment is based on serviceability limit states. However, they cannot provide quantitative information about the loss of effective section that can be used for estimate the strength loss. Consequently, this model is not considered in this work.

4.2 Adopted decay model

On the basis of previous in-lab experimental studies [Viitanen 1996; Viitanen 1991a; Viitanen 1991b], Viitanen et al [Viitanen 2010] developed a model for the decay growth of brown rot in pine sapwood under variant climate conditions. Such a model is divided into two processes: (i) activation process and (ii) mass loss process.

Adopted decay model

A parameter α is used as a relative measure of fungi deterioration activity. α is set initially to 0. Once it reaches the limit value $\alpha=1$, the mass loss initiates. The parameter α varies with time according to:

$$\alpha(t) = \sum_{i=0}^{t} \Delta\alpha(i) \text{ with } \alpha(t) \in [0,1] \qquad (9)$$

where

$$\Delta\alpha(i) = \begin{cases} \dfrac{\Delta t}{t_{crit}(RH(i),T(i))} & \text{if } T(i) > 0°C \\ & \text{and } RH(i) > 95\% \\ -\dfrac{\Delta t}{17520} & \text{otherwise} \end{cases} \qquad (10)$$

where $RH(i)$ and $T(i)$ are the ith air relative humidity (in %) and temperature (ºC), respectively, Δt is the time step between two consecutive climatic records (hours), and t_{crit} (in hours) is estimated as follows:

$$t_{crit}(i) = \left[\frac{2.3T(i) + 0.035RH(i) - 0.024RH(i)T(i)}{-42.9 + 0.14T(i) + 0.45RH(i)} \right] \times 30 \times 24 \qquad (11)$$

Eq. (10) shows that $\Delta\alpha(i)$ increases when $T > 0ºC$ and $RH > 95\%$. Under dry and cold conditions $\alpha(t)$ decreases linearly from 1 to 0 in two years (17520 hours).

Mass loss process

Mass loss (in % of initial weight) occurs once the fungi activation process is reached, ($\alpha(t) = 1$) and it is estimated as:

$$ML(t) = \sum_{i=0}^{t} \left(\frac{ML(RH(i),T(i))}{dt} \times \Delta t \times 1_{\alpha}(i) \right) \qquad (12)$$

where

$$1_{\alpha}(i) = \begin{cases} 0 & \text{if } 0 \leq \alpha(i) < 1 \\ 1 & \alpha(i) = 1 \end{cases} \qquad (13)$$

and

$$\frac{ML(RH(i),T(i))}{dt} = -5.96 \cdot 10^{-2} + 1.96 \cdot 10^{-4} T(i) + 6.25 \cdot 10^{-4} RH(i) \text{ [\% loss/hour]} \qquad (14)$$

According to eq. (10), mass loss only takes place when the temperature is above 0°C and the relative humidity is above 95%. Otherwise mass loss process is stopped.

5 NUMERICAL EXAMPLE

5.1 Problem description

Building roofs are usually made of timber trusses. The design of timber structures requires the verification of a certain number of rules resulting from the codes of practice, such as Eurocode 5 [NF-EN 1995 2005]. Where, these rules should satisfy given requirements related to their ultimate of capacity in ultimate limit state (ULS) and their deformation in service limit state (SLS).

In practice, roof trusses are made and composed of wood members connected by steel plates. Generally, the timber joints are considered completely flexible (free rotations hinges in the connections of the timber members). However, this situation is rarely found in real structures, due to eccentricity and geometrical imperfection of the timber members. Therefore, a rigid joints hypothesis is also considered.

The numerical application is carried out for the roof truss described in Figure 1. The depth b and breadths {h_1, h_2, h_3} of the cross-section of the truss members are optimized by accounting for material and loading uncertainties. Table 1 gives the statistical parameters for the truss parameters, loading and material properties. It is assumed that all random variables follow lognormal distributions.
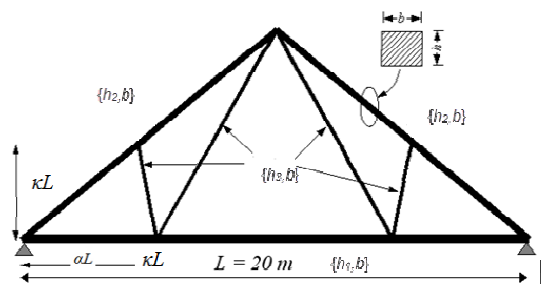
Figure 1:roof truss

For plane timber truss, the ultimate and serviceability limit state functions for each ith member are defined as:

$$\sigma_{m,d}/f_{m,d} + \sigma_{t,0,d}/f_{t,0,d} \leq 1 \quad \text{in tension},$$

$$\left(\frac{\sigma_{m,d}}{k_{crit} \cdot f_{m,d}}\right)^2 + \frac{\sigma_{c,0,d}}{k_{c,z} \cdot f_{c,0,d}} \leq 1 \quad \text{in compression},$$

$$\tau_d/\left(k_\nu f_{\nu,d}\right) \leq 1, \; \delta_{inst}/\delta_{inst,lim} \leq 1 \; (SLS),$$

$$\delta_{fin}/\left(\delta_{fin,lim}\right) \leq 1 \; (SLS)$$

where, $\sigma_{m,d}$, $\sigma_{t,0,d}$, and $\sigma_{c,0,d}$ and are respectively the design values of bending stress, tensile stress along the grain and compressive stress along the grain. $f_{m,d}$, $f_{t,0,d}$, and $f_{c,0,d}$ are respectively the design values of bending strength, tensile strength and the compressive strength along the grain. k_{crit} and $k_{c,z}$ are respectively factors which take into account the reduced bending strength due to lateral buckling and compressive strength due to buckling about the y and z axes in accordance with Eurocode 5. τ_d is the design value of the shear stress and $f_{v,d}$ is the design value of shear strength and k_v is the reduction factor of the shear strength at the notched member. The strength design values are defined by :

$$f_d = k_{mod}\frac{f_k}{\gamma_m} \tag{16}$$

where f_d and f_k are respectively the design and the characteristic values of the strength, k_{mod} is the modification factor, which takes into account the effect of the duration of the load and the moisture content, γ_m is the partial safety factor for a material property. In the deterministic design the values for k_{mod} and γ_m are taken from Eurocode 5.

Table 1: Statistical parameters for materials and loads

Parameter	x_k	x_m	cov
f_m (MPa)	24	33.9	0.2
f_c (MPa)	21	29.66	0.2
f_t (MPa)	14	19.77	0.2
$f_{c,90}$ (MPa)	2,5	3.53	0.2
f_v (MPa)	4	5.65	0.2
E (GPa)	10.78	11	0.2
Permanent load (kN/m²)	620	466.52	0.2
Snow (kN/m²)	1193	798.81	0.3
Wind (kN/m²)	1320	883.53	0.36

Eq. (11) considers the following stress criteria: i) tension and bending: where the tension is parallel to the grain; ii) compression: where members are checked for compressive strength as well as for buckling; iii) shear: for all the truss members; and iv)

deformation: corresponding to the serviceability state functions, where δ_{inst} and δ_{fin} are respectively the instantaneous deflection and the final deflection composed with the instantaneous and creep deflections. $\delta_{inst,lim}$ and $\delta_{fin,lim}$ are respectively the limit values for instantaneous and final deflections, taken respectively to $L_i/300$ and $L_i/200$ in mm for the ith component.

To find the optimal design that minimizes the structural volume of the roof truss, three optimization methods are applied:

- The DDO method on the basis of the safety factors prescribed by the Eurocode 5.
- The RBDO method using the SORA approach without considering degradation model.
- The TD-RBDO method using the SOTVRA approach considering degradation model.

The target reliability index for one year for the ultimate limit state is set to 3.8 and for serviceability limit state is set to 2.9. In the TD-RBDO method, the target reliability for allowable lifetime T_L fixed to 30 years is estimated with Eq. (4). For all the optimal solutions found by these methods, a time-dependent reliability analysis considering the decay model is performed.

Timber decay is assessed by considering the deterioration model presented in section 4. Such a model depends mainly on specific environmental conditions of a given place and it allows for estimating the mass loss during the time. For illustrative purposes, it is considered that the roof was exposed to a very humid environment corresponding to the city of Nantes in France. Hourly variations of temperature and relative humidity for 30 years [1980-2010] were used in the example. Nantes is close to the Atlantic Ocean in the Northern part of the country and has a temperate oceanic climate with annual mean temperature and relative humidity of 12.7 ºC and 81%, respectively.

5.2 Results

Tables 2 and 3 present the optimal solutions corresponding to different optimization methods considering flexible and rigid joints, respectively. It is observed that the optimized volume obtained by TD-RBDO doubles the values calculated by the other approaches. This is expected because DDO and RBDO did not include directly the deterioration process. Small volumes are found when the design considers rigid joints.

Table 2: Design optimization results for flexible joints

Parameter	DDO	RBDO	TD-RBDO
b (mm)	166.89	175.22	240.30
h_1 (mm)	235.81	248.80	375.32
h_2 (mm)	333.78	350.44	480.60
h_3 (mm)	222.52	233.62	320.40
Volume (cm³)	1182.65	1305.4	2524.11

Table 3: Design optimization results for rigid joints

Parameter	DDO	RBDO	TD-RBDO
b (mm)	162.24	143.80	236.27
h_1 (mm)	216.24	191.74	315.03
h_2 (mm)	324.50	287.61	472.55
h_3 (mm)	216.33	191.74	315.03
Volume (cm³)	1100.12	864.26	2333.04

Figure 2 depicts the time-dependent reliability indexes of the optimal design of the roof truss given by the DDO approach. The initial reliability index at $t = 0$ of the ultimate limit state satisfies the target reliability index of one year. However, the initial reliability index at $t = 0$ of the serviceability limit state is not checked. In other words, the use of the partial safety factors in the design optimization cannot guarantee the target reliability. When timber degradation is considered, the reliability indexes of the serviceability and ultimate limit state fell suddenly, where the serviceability reliability index reaches the target after 8 years when the flexible joints are considered. This is explained by the fact that the structure is less rigid with flexible joints increasing the critical deflections.
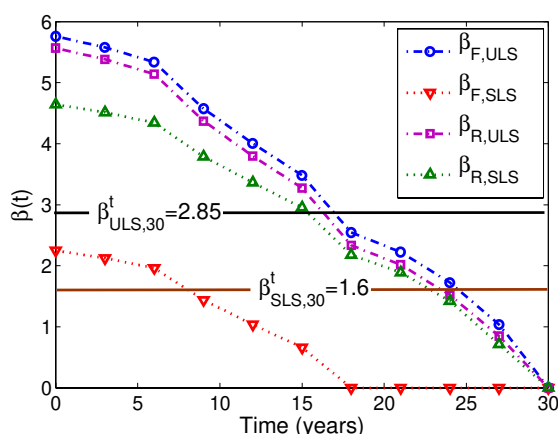

Figure 2: Time-dependent reliability index of DDO solution for flexible (F) and rigid (R) joints.

Figure 3 shows the time-dependent reliability profiles of the RBDO solution, where this optimal solution is estimated without the decay model and by considering only the target reliability of one year. Figure 3 indicates that the reliability indexes for ultimate and serviceability limit states at the initial time are checked regarding the target values for one year (3.8 for ULS and 2.9 for SLS). However, considering the timber decay, the reliability indexes decrease quickly and the serviceability and ultimate target reliability indexes are reached after 13 and 9 years respectively for the rigid joints.
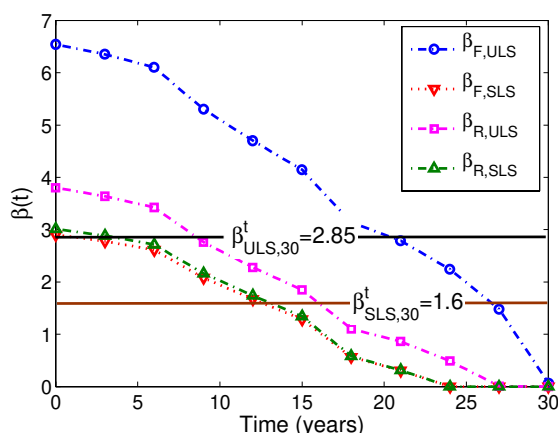

Figure 3: Time-dependent reliability index of RBDO solution for flexible (F) and rigid (R) joints

Figure 4 indicates that the TD-RBDO method combined with the SOTVRA approach give the optimal design that satisfies the serviceability and reliability requirements during the 30 years lifetime. However, the proposed design solution is more expensive. The optimal volume is about 2.7 times larger than the RBDO volume (Tables 2 and 3). This large volume ensures the reliability and serviceability requirements under a very humid exposure that accelerates timber decay. Different results could be obtained for other climates. Besides, if the failure cost is considered in the RBDO problem, the total cost of the TD-RBDO may be lower. For all cases, the target reliability is reached early when rigid joints are considered because these joints generate additional bending moments in the truss.
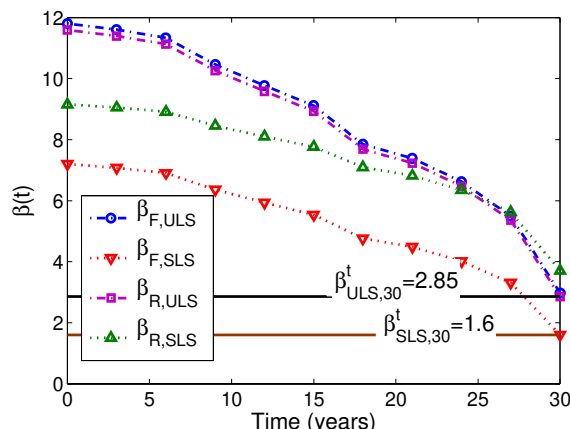

Figure 4: Time-dependent reliability index of TD-RBDO solution for flexible (F) and rigid (R) joints.

6 CONCLUSIONS

This study focused on the design optimization of timber truss structures by accounting for uncertainties, climate variations, and serviceability and safety constraints. The preliminary results indicate that the TD-RBDO solution ensures the serviceability and reliability requirements during the whole lifecycle. However, the optimized solution needs a large material volume (larger construction costs) in comparison with DDO and RBDO for the environmental exposure. Further work in this area will focus on: (1) the consideration of climate uncertainties, (2) the design optimization for other locations, and (3) the consideration of real costs (including the failure cost).

7 ACKNOWLEDGMENTS:

8 REFERENCES

[Aoues 2009] Aoues, Y.; Bastidas-Arteaga, E.; Chateauneuf, A. Optimal design of corroded reinforced concrete structures by using time-variant reliability analysis, in: Furuta, H.; Frangopol, D. M.; Shinozuka, M. (eds.): 10th International Conference on Structural Safety and Reliability ICOSSAR, Osaka, Japan, pp. 1580–87.

[Aoues 2010] Aoues, Y.; Chateauneuf, A. Benchmark study of numerical methods for reliability-based design optimization, in: Structural and multidisciplinary optimization, Vol. 41, No. 2, pp. 277–294.

[Brischke 2014] Brischke, C.; Thelandersson, S. Modelling the outdoor performance of wood products – A review on existing approaches, in: Construction and Building Materials, Vol. 66, pp. 384–397.

[Du 2004] Du, X.; Chen, W. Sequential optimization and reliability assessment method for efficient probabilistic design, in: Journal of Mechanical Design, ASME, Vol. 126, No. 2, pp. 225–233.

[EN 252 1990] EN 252 Wood preservatives. Field test methods for determining the relative protective effectiveness in ground contactitle, European Committee for Standardization.

[De Freitas 2010] De Freitas, R. R.; Molina, J. C.; Calil Junior, C. Mathematical model for timber decay in contact with the ground adjusted for the state of São Paulo, Brazil, in: Materials Research, Vol. 13, No. 2, pp. 151–158.

[Isaksson 2012] Isaksson, T.; Brischke, C.; Thelandersson, S. Development of decay performance models for outdoor timber structures, in: Materials and Structures, Vol. 46, No. 7, pp. 1209–1225.

[Kravanja 2013] Kravanja, S.; Turkalj, G.; Šilih, S.; Žula, T. Optimal design of single-story steel building structures based on parametric MINLP optimization, in: Journal of Constructional Steel Research, Vol. 81, pp. 86–103.

[Leicester 2009] Leicester, R. H.; Wang, C.; Nguyen, M. N.; MacKenzie, C. E. Design of Exposed Timber Structures, Vol. 9, No. 3, p. 217.

[Lourenço 2012] Lourenço, P. B.; Sousa, H. S.; Brites, R. D.; Neves, L. C. In situ measured cross section geometry of old timber structures and its influence on structural safety, in: Materials and Structures, Vol. 46, No. 7, pp. 1193–1208.

[NF-EN 1990 2003] NF-EN 1990 Eurocode 0: Basis of structural design Final draft prEN 1990-, Brussels, European Comitee for Standardization.

[NF-EN 1995 2005] NF-EN 1995 Eurocode 5: design of timber structure, Part 1-1 General rules and rules for buildings, Final draft prEN 1995-1-1, Brussels, European Comitee for Standardization.

[Ryan 2014] Ryan, P. C.; Stewart, M. G.; Spencer, N.; Li, Y. Reliability assessment of power pole infrastructure incorporating deterioration and network maintenance, in: Reliability Engineering & System Safety, Vol. 132, pp. 261–273.

[Šilih 2005] Šilih, S.; Premrov, M.; Kravanja, S. Optimum design of plane timber trusses considering joint flexibility, in: Engineering Structures, Vol. 27, No. 1, pp. 145–154.

[Sousa 2014] Sousa, H. S.; Branco, J. M.; Lourenço, P. B. Characterization of Cross-Sections from Old Chestnut Beams Weakened by Decay, in: International Journal of Architectural Heritage, Vol. 8, No. 3, pp. 436–451.

[Tomás 2010] Tomás, A.; Martí, P. Shape and size optimisation of concrete shells, in: Engineering Structures, Vol. 32, No. 6, pp. 1650–1658.

[Tran 2014] Tran, V.-D.; Oudjene, M.; Méausoone, P.-J. FE analysis and geometrical optimization of timber beech finger-joint under bending test, in: International Journal of Adhesion and Adhesives, Vol. 52, pp. 40–47.

[Viitanen 1996] Viitanen, H. Factors affecting the development of mould and brown rot decay in wooden material and wooden structures. Effect of humidity, temperature and exposure time.PhD Dissertation, University of Uppsala.

[Viitanen 1991a] Viitanen, H.; Ritschkoff, A.-C. Brown rot decay in wooden con- structions. Effect of temperature, humidity and moisture, The Swedish University of Agricultural Sciences, Department of Forest Products, Report no 222, Uppsala.

[Viitanen 1991b] Viitanen, H.; Ritschkoff, A.-C. Mould growth in pine and spruce sapwood in relation to air humidity and temperature, The Swedish University of Agricultural Sciences, Department of Forest Products, Report no 221, Uppsala.

[Viitanen 2010] Viitanen, H.; Toratti, T.; Makkonen, L.; et al. Towards modelling of decay risk of wooden materials, in: European Journal of Wood and Wood Products, Vol. 68, No. 3, pp. 303–313.