# Bending-torsional behavior analysis using a refined beam theory

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Abstract: In this paper, the bending-torsional behavior analysis of a functionally graded material (FGM) cantilever beam (I-section) was studied using a high-order theory, called refined beam theory (RBT) based on the 3D solution of Saint-Venant(SV). This theory takes into account the effects of edges to describe a larger three-dimensional solution inside the beam. The model includes a set of deformations that reflect the mechanical behavior of the FGM beam, in order to describe a larger three-dimensional solution inside the beam (Poisson's effects, out-of-plane warpings, and distortions). These deformations are related to both the shape and materials of the section. To study this beam and apply this theory, an easy-to-use tool called Cross-Section and Beam analysis (CSB) was used. The Young's modulus of an FGM cantilever beam varies through-thickness by the power-law distribution. The results obtained were compared with those obtained with the 3D/SV solution.

**Mots-clefs :** FGM, bending, torsional, power-law, Young's modulus

## I. INTRODUCTION

In recent years, a new class of Composite materials has emerged of advanced activity and use called Functionally graded materials (FGM), by a group of Japanese researchers at the National Space Laboratory (STA). The FGM is among the promising candidates in various engineering sectors and industries, such as automobile, aerospace, defense industries, medicine sectors, and aircraft. These gradient materials with different properties were developed in order to withstand thermal and mechanical stresses through the changing microstructure gradually and continuously through the thickness to improve its performance. Usually, these materials are composited from a mixture of metal and ceramic. The difficulty is in analyzing the FGM beam through understanding and predicting its mechanical behavior especially in the case of sections (open, closed, thin-walled) are embedded. This calls for the development of a suitable beam theory for beam analysis that takes into account the three-dimensional effects when using a one-dimensional model with cross-section deformation.

through Euler Bernoulli's theory, a beam model was derived to study the nonlinear bending behavior of a two-dimensional FGM beam by Li and Hu (2018). Hebbar et al (2020) used shear deformation theories to study the bending, buckling, and vibration of FGM beams. Kadoli et al (2008) used high-order shear deformation theory to study the static behavior and apply the displacement field of the FGM beam. Ying et al (2008) presented the theory of two-dimensional elasticity in order to study the vibration and bending of the FGM beam. Khebizi et al. (2019) studied the mechanical behavior of FGM beam, through the 3D Saint-theory Venant's model involving Poisson's effects and out-of-plane warping.

In this paper we based on the RBT theory built on the SV solution in order to study the mechanical behavior of the FGM cantilever beam (I - section); this beam is subjected to the torsional-bending introduced in the free side by three loads applied in different places. The model includes various deformations that reflect the behavior of the FGM beam (Poisson's effects, out-of-plane warpings, and distortions).

## II. Functionally graded material

the distribution of the power-law (P–FGM); Young's modulus is given by Khebizi et al. (2019), see Eq. (1). For the exponential distribution (E–FGM), Young's modulus is given by Khebizi et al. 2019, see Eq. (2).

## III. The beam problem

1. **Saint Venant Beam Theory** (see article of khebizi et al (2019)).
2. **Refined Beam Theory** (see article El Fatmi (2016)).

## VI. Numerical results and discussions

By studying the FGM cantilever beam (I-section) (L = 1.2 m), the most important results obtained based on the three-dimensional displacement fields are presented. This beam is subject to bending-torsional behavior resulting from identical loads (1KN per force) applied at the free end of the beam (see Fig. 1). The beam is composed of aluminum (AL: Em = 70 GPa, v = 0.3); and Zirconia (Zero2: Ec= 200 GPa, v = 0.3) These characteristics differ through the thickness of the beam according to different styles show in Fig. (1.2). The top surface of the beams contains pure aluminum (y =+ *h*/2), while the bottom surface contains pure zirconia (y = − *h*/2).

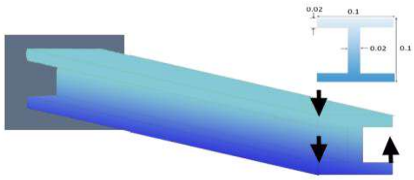


FIGURE 1. The equilibrium beam problem.

1. Cross-section analyses

The model includes various deformations that reflect the behavior of the FGM beam. Fig.2 shows the Poisson's effects (red color)related to the axial force (N) and the bending moments($M_x$, $M_y$), and the out of plane warpings(blue color) related to the shear forces ($T_x$, $T_y$) and the torsional moment($M_t$). Moreover. Fig. 3 shows 15 additional deformation modes, 10 in-plane (pink color) and 5 outside (blue color) for the cross-section (I-section) of FGM at P=1.

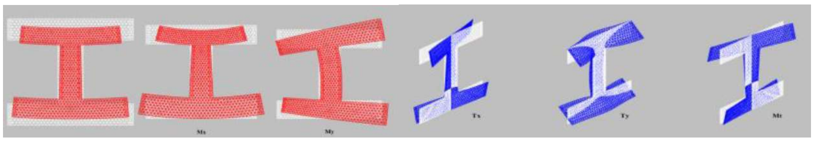


FIGURE 2. Cross-sections deformations: Poisson's effects and warpings for the FGM sections P=1.

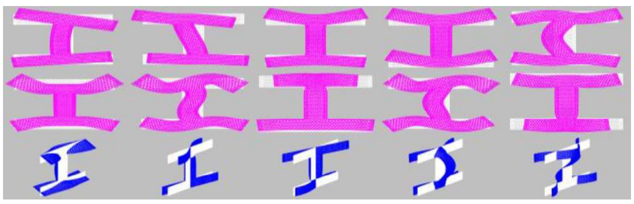


FIGURE 3. Cross-section deformations: distortions for the FGM sections P=1.

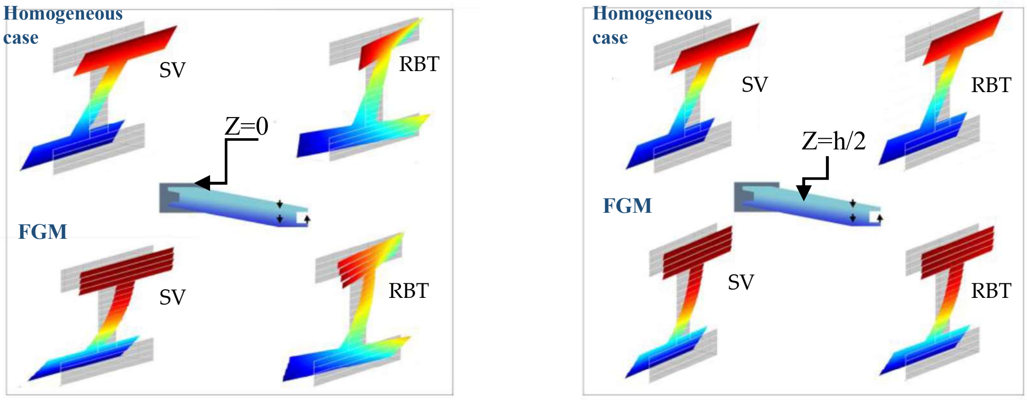
2. Analysis of the bending-torsion cantilever beam

Table.1 Shows the maximum torsional rotation for the FGM cantilever beam according to RBT and SV, we can see the remarkable difference between the two theories. We conclude from Tables 1 that the behavior of RBT theory is stiffer than SV theory.

TABLEAU 1. The Maximum torsional rotation for FGM cantilever beam according to RBT and SV (rad).

| **power-low expanate** | **RBT** | **SV** |
|---|---|---|
| full metal | 8,1096 | 9,2776 |
| P=0,5 | 4,7417 | 5,3371 |
| P=2 | 3,9397 | 4,4434 |
| P=5 | 3,6008 | 4,0708 |
| full ceramic | 2,383 | 3,24471 |

Fig. 4 shows a comparison between the axial stress domains when embedding and in the Intermediate Level of the beam for both the homogeneous cases and the FGM. we can be seen (Z=0) that the axial stress is linear in the homogeneous cases of the SV solution due to the bending stress. In the case of FGM the axial stresses are nonlinear due to the difference in Young's modulus across beam thickness. While the results for the axial stresses are totally different for the RBT than those obtained by the SV solution, That's result of the bending stress and those induced by the constrained warping of torsion. far from the embedding at Z=h/2 the axial stresses of the RBT are compatible with the SV's solution. Fig 5. shows axial stress variations along the FGM beam for two points (A and B) belonging to the upper flange. The results are obtained by the following boundary condition: deformable section for RBT theory and non-deformable for SV. we can be observing a large difference between the axial stress results of the two theories at the embedding level, while the results converge away from the embedding until the two theories are similar at the mean plane of the FGM beam. Whereas the section is non-deformable for the RBT theory the results are similar to the SV solution. we can conclude that the RBT theory takes into account the edge effects in order to better describe the behavior of the FGM beam, and according to the SV solution, it describes the exact solution within the beam.




(a) Axial stress at Z=0 (b) Axial stress at Z=h/2

FIGURE 4. Comparison of the axial stresses distributions at Embedding (Z=0) and Intermediate Level Z=h/2 obtained by SV and RBT

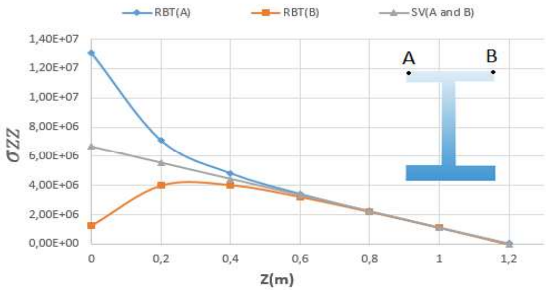



FIGURE 5. axial stresses variations along the FGM beam for two points (A and B) belonging to the upper flange. Comparison of SV and RBT

**Conclusions**

In this paper, the bending-torsional behavior of the FGM cantilever beam is studied, this study based on refined beam theory (RBT) built on 3D solution of Saint-Venant (SV). The main deformation modes are presented by enriching the displacement model (Poisson's effects, out-of - plane warpings, and distortions) extracted from the Saint-Venant (SV) solution, wherein the theory leads to a beam theory suited to the nature of the section in terms of shape and materials. Through the obtained results, we mention the most important differences between RBT / SV theory and SV theory:

- A good description of Edge effects by RBT theory. As for the Saint-Venant solution, it is valid far from the edges.
- Away from the edges, the solution to the RBT approaches Saint-Venant's solution.
- The neglected twisting of the Saint-Venant solution generates additional pivotal pressures for the RBT, resulting in completely different behavior than the Saint-Venant solution.
- The stress of the axial fields is quite different for the two theories (shape and values) upon embedding.

## REFERENCES

Li, L., Li, X., & Hu, Y. (2018). Nonlinear bending of a two-dimensionally functionally graded beam. Composite Structures, 184, 1049-1061. doi.org/10.1016/j.compstruct.2017.10.087

Hebbar, N., Hebbar, I., Ouinas, D., & Bourada, M. (2020). Numerical modeling of bending, buckling, and vibration of functionally graded beams by using a higher-order shear deformation theory. *Frattura ed Integrità Strutturale*, *14*(52), 230-246. DOI: 10.3221/IGF-ESIS.52.18

Kadoli, R., Akhtar, K., & Ganesan, N. (2008). Static analysis of functionally graded beams using higher order shear deformation theory. Applied Mathematical Modelling, *32*(12), 2509-2525. doi: 10.1016/j.apm.2007.09.015

Ying, J., Lü, C. F., & Chen, W. Q. (2008). Two-dimensional elasticity solutions for functionally graded beams resting on elastic foundations. Composite Structures, *84*(3), 209-219. doi: 10.1016/j.compstruct.2007.07.004

Khebizi, M., Guenfoud, H., Guenfoud, M., & El Fatmi, R. (2019). Three-dimensional modelling of functionally graded beams using Saint-Venant's beam theory. *Structural Engineering and Mechanics*, *72*(2), 257-273. DOI: https://doi.org/10.12989/sem.2019.72.2.257

El Fatmi, R. (2016). A refined 1D beam theory built on 3D Saint-Venant's solution to compute homogeneous and composite beams. *Journal of Mechanics of Materials and Structures*, *11*(4), 345-378. doi.org/10.2140/jomms.2016.11.345