Probabilistic analysis based on combination of polynomial chaos and smart truncation schemes: application to fatigue crack growth.

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ABSTRACT This work presents an original uncertainty propagation method, called sparse-PCE, developed to assess the reliability of a cracked plate with spatially varying uncertain mechanical properties. It combines regression techniques to compute the unknown coefficients of the PCE-based metamodel and an efficient truncation scheme which uses prior available second order statistical moment information to identify the most important components of the polynomial chaos basis on the model responses of interest. In this way, the PCE coefficients corresponding to the components with weak effects are discarded, and the computational efforts devoted to solving the regression problem is significantly reduced. An economy index is introduced in the form of a ratio between the respective cardinalities of the sparse and the full chaos polynomial basis, which allows us to objectively assess the computational cost saving obtained by the proposed truncation scheme based on second moment information.

Keywords random field, uncertainties propagation, moments analysis, sensitivity analysis, high probabilistic dimension, polynomial chaos, fatigue crack growth

I. INTRODUCTION

Fatigue crack growth is a random process (Ghonem and Dore, 1987), mainly due to the uncertainties observed on the mechanical properties of the materials, on the applied loading as well as the parameters defining the geometry of the structure. These sources of uncertainties may have a harmful effect on the integrity of cracked structures and should be taken into account to ensure safe designs. To this end, uncertainties propagation methods wich have been developed over the past forty years (Stefanou, 2009), seems to be the best alternative. Among them we find the well-established metamodeling method based on Polynomial Chaos Expansion (PCE), introduced to deal with engineering problems in the early 1990's thanks to the work of (Ghanem and Spanos, 1991). The key idea is to build an accurate mathematical approximation of the model response of interest (e.g. fatigue lifetime) based on a limited set of evaluations of the primary implicit mechanical model. Such an approximation is referred to as a response surface, surrogate model or metamodel. Once the PCE-based metamodel is obtained, different kinds of uncertainty propagation analysis, such as reliability analysis, can be carried out by simply performing the well-known Monte-Carlo Simulations (MCS). Unfortunately, the PCE-based metamodeling method still suffer from inefficiency when dealing with problems having high probabilistic dimensionality defined as the number of the uncertain parameters.

II. Proposed approach

A. *Construction of PCE-based metamodels*

Let us consider a computational model f describing the behavior of an engineering system, whose input parameters $\boldsymbol{x} = \{x_1, \dots, x_N\}^T \in \mathfrak{D}_{\boldsymbol{x}}$ are uncertain quantities represented by an N-dimensional random variable $\boldsymbol{X} = \{X_1, \dots, X_N\}^T \in \mathfrak{D}_{\boldsymbol{X}}$ with a prescribed probability density function $p_{\boldsymbol{X}}(\boldsymbol{x})$, and $y = f(\boldsymbol{x})$ the model response of interest taken for the sake of simplicity as a scalar. Let also assume that the random variable Y with a probability density function $p_Y(y)$, representing the variability of the model response y induced by the randomness of the input parameters, has a finite variance, and that the components of the N-dimensional random variable $\boldsymbol{X} = \{X_1, \dots, X_N\}^T$ are statistically independent. The PCE-based metamodel of $Y = f(\boldsymbol{X})$ thus reads (Xiu and Karniadakis, 2002):

$$Y = f(\boldsymbol{X}) \approx f^{PCE}(\boldsymbol{X}) = \sum_{k=0}^{P-1} a_k \, \boldsymbol{\Psi}_{\boldsymbol{\alpha}_k}(\boldsymbol{X}) \tag{1}$$

where P denotes the number of terms in the PCE, $\boldsymbol{\alpha}_k = (\alpha_k^1, \dots, \alpha_k^N), k = 0, \dots, P-1$ a set of multi-indices also called N-tuples of integers (i.e., $\boldsymbol{\alpha}_k \in \mathbb{N}^N$), $\boldsymbol{\Psi}_{\boldsymbol{\alpha}_k}, k = 0, \dots, P-1$ a set of multivariate orthonormal polynomials with respect to $p_{\boldsymbol{X}}(\boldsymbol{x})$, whose total degree $|\boldsymbol{\alpha}_k| = \alpha_k^1 + \dots + \alpha_k^N$ and $a_k, k = 0, \dots, P-1$ a set of real valued deterministic coefficients to be determined.

The size of the PCE-based metamodel given by equation (1), that is the number of terms P retained in the summation, can be determined by following one of the truncation schemes available in the PCE literature (Blatman, 2009). The most used one consists in retaining the terms corresponding to multivariate polynomials $\boldsymbol{\Psi}_{\boldsymbol{\alpha}_k}, k = 0, \dots, P-1$ whose total degrees $|\boldsymbol{\alpha}_k| = \alpha_k^1 + \dots + \alpha_k^N, k = 0, \dots, P-1$ do not exceed a prescribed degree p, chosen to ensure a better accuracy of the metamodel. Based on this rule, the number of terms P in the truncated PCE is given by:

$$P = \frac{(p+N)!}{p!\,N!} \tag{2}$$

Equation (2) clearly shows that the number of terms in the PCE grows exponentially with N, which could induce an unaffordable computational cost in the determination of the unknown coefficients when dealing with uncertainty propagation problems with a high probabilistic dimensionality and especially when the corresponding physical model is itself computationally time-demanding.

In engineering problems, the components of the N-dimensional random variable $\boldsymbol{X} = \{X_1, \dots, X_N\}^T$ may have different distributions. Thus, the use of isoprobabilistic transformations $\boldsymbol{X} = T(\boldsymbol{U})$. Then, the PCE-based metamodel represented by equation (1) can be naturally rewritten in the standard random space as follows:

$$Y = f(\boldsymbol{X}) = f \circ T(\boldsymbol{U}) = h(\boldsymbol{U}) \approx h^{PCE}(\boldsymbol{U}) = \sum_{k=0}^{P-1} a_k \, \boldsymbol{H}_{\boldsymbol{\alpha}_k}(\boldsymbol{U}) \tag{3}$$

B. Computation of the PCE coefficients by regression

Regression methods have been used first by (Isukapalli, 1999) and later by (Berveiller, 2005), to compute the unknown coefficients of the PCE. Unlike projection methods, where the PCE coefficients are computed one by one by evaluating multidimensional integrals, regression methods estimate all the coefficients at the same time by solving a minimization problem in the least-squares sense, which could considerably reduce the computation effort. The regression technique consists in finding the vector of coefficients $\boldsymbol{a}$ that minimizes the mean square error $\mathbb{E}[(\boldsymbol{a}^T\ \boldsymbol{\mathcal{H}}(\boldsymbol{U}) - f \circ T(\boldsymbol{U}))^2]$, that is:

$$\hat{\boldsymbol{a}} = \underset{\boldsymbol{a} \in \mathbb{R}^P}{\operatorname{argmin}} \{A(\boldsymbol{a}) \equiv \mathbb{E}[(\boldsymbol{a}^T\ \boldsymbol{\mathcal{H}}(\boldsymbol{U}) - f \circ T(\boldsymbol{U}))^2]\} \tag{4}$$

In practice, the minimization problem defined by equation (4) is discretized on the basis of a set of sample points $\boldsymbol{\mathcal{U}} = \{\boldsymbol{u}^j = (u_1^j, \dots, u_N^j), j = 1, \dots, M\}$, also called experimental design, to replace the expectation operator $\mathbb{E}[.]$ by its empirical estimate. Thus, the minimization problem reads:

$$\hat{\boldsymbol{a}} = \underset{\boldsymbol{a} \in \mathbb{R}^P}{\operatorname{argmin}} \left\{ \frac{1}{M} \sum_{j=1}^{M} (h^{PCE}(\boldsymbol{u}^j) - f \circ T(\boldsymbol{u}^j))^2 \right\} = \underset{\boldsymbol{a} \in \mathbb{R}^P}{\operatorname{argmin}} \left\{ \frac{1}{M} \sum_{j=1}^{M} \left(\sum_{k=0}^{P-1} a_k\, \boldsymbol{H}_{\alpha_k}(\boldsymbol{u}^j) - f \circ T(\boldsymbol{u}^j) \right)^2 \right\} \tag{5}$$

where $h^{PCE}(\boldsymbol{u}^j)$ and $f \circ T(\boldsymbol{u}^j)$ are respectively the responses of the PCE-based metamodel and the primary mechanical model at the point $\boldsymbol{u}^j$.

The choice of a suitable experimental design $\boldsymbol{\mathcal{U}} = \{\boldsymbol{u}^j = (u_1^j, \dots, u_N^j), j = 1, \dots, M\}$ is of great importance, especially its size M, to obtain a well-conditioned regression problem and consequently accurate estimates of the PCE coefficients. Indeed, if M is just slightly greater than the number P of unknown coefficients to be computed, this may lead to an ill-conditioned information matrix $\boldsymbol{\mathcal{H}}$ and consequently to an intractable regression problem. On the other hand, that is, if M is very high, this may induce an unaffordable computational cost in case the mechanical model itself is computational time-demanding, since the corresponding number of evaluations of the mechanical model will be high. In the literature, the value of M is commonly chosen in the range [$2P$, $3P$] to ensure a better balance between the computational cost and the accuracy of the estimates. In this paper the parameter M is obtained by a smart truncation scheme combining low-order interactions terms and prior available information on second order moment.

C. Truncation scheme based on low-order interactions

For problems with high dimensionality N, a major part of the PCE coefficients represents interactions between uncertain parameters, even for moderate truncation degree p. Fortunately, for engineering problems experience has shown that high order interactions have often insignificant effect, which means that the corresponding PCE coefficients are close to 0. Thus, the size of the polynomial chaos basis can be reduced by retaining only the terms representing main and low-order interactions effects.

Let $\mathcal{H}^p = \left\{ \boldsymbol{H}_{\alpha_k}, \boldsymbol{\alpha}_k \in \mathbb{N}^N \text{ such that } \sum_{i=1}^{N} \alpha_k^i \leq p \right\}$ a complete polynomial chaos basis for a given truncation degree p.

$\mathcal{H}^{p,q} = \left\{ \boldsymbol{H}_{\alpha_k}, \boldsymbol{\alpha}_k \in \mathbb{N}^N \text{ such that } \sum_{i=1}^{N} \alpha_k^i \leq p \text{ and } \sum_{i=1}^{N} \mathbb{1}_{\{\alpha_k^i \neq 0\}} \leq q \right\}$ an incomplete, called also sparse, polynomial chaos basis for a given truncation degree p and interaction order $q < p$, i.e., only q-variate polynomials whose respective total degrees do not exceed a given degree p are retained. If the allowed maximum interaction order q is not high, the cardinality of the sparse polynomial chaos basis $\mathcal{H}^{p,q}$ will be much lower than that of the complete polynomial chaos basis $\mathcal{H}^p$.

The efficiency of the truncation scheme based on sparse polynomial basis can be assessed by the economy $\mathcal{E}^{p,q}$ defined by the following ratio:

$$\mathcal{E}^{p,q} = \frac{\text{card}(\mathcal{H}^p) - \text{card}(\mathcal{H}^{p,q})}{\text{card}(\mathcal{H}^p)} \times 100 \quad (6)$$

where $\text{card}(\mathcal{H}^p)$ and $\text{card}(\mathcal{H}^{p,q})$ are the cardinalities of the complete $\mathcal{H}^p$ and sparse $\mathcal{H}^{p,q}$ polynomial chaos bases respectively.

The maximum interaction order q can be chosen either by following a step-by-step scheme where the value of q is increased gradually to achieve a target level of accuracy on the estimates of the PCE coefficients, or by performing a preliminary screening analysis (Morris, 1991) which allows, based statistical analysis of a set of local gradients , to split the uncertain parameters into three categories, those with weak main effect, those with linear and additive effects and those with nonlinear or interaction effect. Note that screening analysis are not computational time-demanding, thus the loss of efficiency on the whole computational process is very limited.

D. Truncation scheme based on second moment information

If a prior information about the estimate of the second order statistical moments is already available, the latter could be a useful tool to identify the most significant terms on the quantities of interest, when a step-by-step algorithm is used to build the polynomial chaos basis. Indeed, at each iteration k of this algorithm, the polynomial chaos basis - denoted here by $\mathcal{H}^{p,q,\sigma^2}$ (i.e., σ^2 in $\mathcal{H}^{p,q,\sigma^2}$ refers to the target variance of the quantity of interest) - is enriched by a new candidate polynomial $\boldsymbol{H}_{\alpha_k}$. If the related PCE term induces a significant change on the estimate of the variance $\sigma^2_{PCE,k}$, thus getting closer to the target variance σ^2, the candidate $\boldsymbol{H}_{\alpha_k}$ is retained. Otherwise, i.e. the relative error $\left| \sigma^2_{PCE,\,k} - \sigma^2_{PCE,\,k-1} / \sigma^2_{PCE,\,k-1} \right|$ is smaller than ε_1, the candidate $\boldsymbol{H}_{\alpha_k}$ is discarded from the polynomial chaos basis and another candidate is tested in the next iteration until a given level of accuracy ε_2 is achieved for the whole iterative procedure.

Note that the values of ε_1 used in the criterion of enrichment of the polynomial chaos basis and ε_2 used in the stopping condition of the step-by-step algorithm, are set respectively to 10^{-6} and 10^{-3}, which allow us, on the one hand, to avoid ill-conditioned information matrix, thus an intractable regression problem, and, on the other hand, to ensure a good accuracy on the estimates of the quantities of interest. Of course, other values can be chosen depending on the complexity of the problem of interest and the accuracy to be achieved.

III. Application to fatigue crack growth

The purpose of this section is to study the efficiency and accuracy of the method developed previously based on a mechanical problem of fatigue crack growth from the work of (Long and al, 2016). We consider a rectangular plate of height $2L = 2\ units$ and width $W = 1\ unit$ visualized in figure 1. It is subjected to tensile load $\sigma = 1\ unit$ on its bottom and top edges and has an open inclined crack with dimensions $a = z = 0.5\ unit$. Due to the orientation of the initial crack with respect to the applied load, this later naturally tends to propagate in a mixed fracture mode, instead of a simple opening fracture mode. Thus, a FEM is developed in the software (cast3m, 2021) to compute the fracture driving forces, namely the opening fracture mode SIF K_I, the in-plane shear fracture mode SIF K_{II} and the bifurcation angle θ.

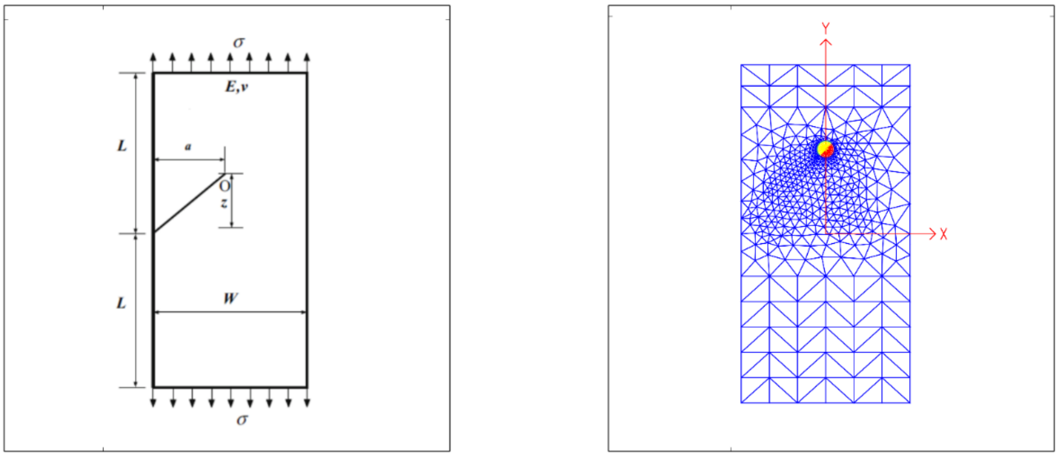

Figure 1 Inclined edge-cracked plate: geometry and applied loads (left), finite element mesh (right)

The Young's modulus $E(\mathbf{z},\omega)$ of the constitutive material of the cracked plate is considered as an uncertain parameter whose variability varies along both the horizontal and the vertical directions denoted by x and y, respectively, and gathered in the vector $\mathbf{z} = (x,y)$. It is modeled by a two-dimensional lognormal random field, with mean value $\mu_E = 20.7\ 10^6\ units$ and standard deviation $\sigma_E = 2.07\ 10^6\ units$, which can be defined simply as the exponential of a normal random field $v(\mathbf{z},\omega) = \mathrm{Ln}(E(\mathbf{z},\omega))$ with mean $\mu_v = \ln(\mu_E) - \frac{1}{2}\ln(1 + \sigma_E^2/\mu_E^2)$ and standard deviation $\sigma_v = \sqrt{\ln(1 + \sigma_E^2/\mu_E^2)}$:

$$E(\mathbf{z},\omega) = \exp[v(\mathbf{z},\omega)] = \exp[\mu_v + \sigma_v . u(\mathbf{z},\omega)] \quad (7)$$

where ω is a parameter to underline the randomness of $E(\mathbf{z},\omega)$ and $u(\mathbf{z},\omega)$ is a standard normal random field of zero mean and unit standard deviation, governed by the following exponential autocorrelation function:

$$\rho(\mathbf{z}_1, \mathbf{z}_2) = \exp\left[-(\frac{|x_1 - x_2|}{l_{cx}} + \frac{|y_1 - y_2|}{l_{cy}})\right] \tag{8}$$

The standard normal random field $u(\mathbf{z}, \omega)$ is discretized using the Karhunen-Loève (KL) method (Ghanem and Spanos, 1991).

$$\hat{E}(\mathbf{z}, \omega) = \exp[\mu_v + \sigma_v . \sum_{i=1}^{M} \sqrt{\lambda_i}\, \varphi_i(\mathbf{z})\, u_i(\omega)] \tag{9}$$

where $u_i(\omega), i \in \{1, \dots, M\}$ are independent standard normal variables, λ_i and $\varphi_i(\mathbf{z})$ are respectively eigenvalues and eigenfunctions obtained by solving the following Fredholm integral equation corresponding to the autocorrelation function $\rho(\mathbf{z}, \mathbf{z}')$:

$$\int_{\Omega} \rho(\mathbf{z}, \mathbf{z}')\, \varphi_i(\mathbf{z}') = \lambda_i\, \varphi_i(\mathbf{z}) \tag{10}$$

Fortunately, for our problem where the two-dimensional spatial domain $\Omega = [-0.5, 0{,}5] \times [-1, 1]$ has a rectangular geometry and the random field $E(\mathbf{z}, \omega)$ follows an exponential autocorrelation function, the Fredholm integral equation can be solved analytically and a closed from solutions of the eigenvalues λ_i and the eigenfunctions $\varphi_i(\mathbf{z})$ can be obtained.

In the following, a 24th order truncated KL expansion is used to model the spatial variability of the Young's modulus of the constitutive material of the cracked plate following a lognormal random field. This means that only the first 24 largest eigenvalues λ_i, already sorted in ascending order, and the corresponding eigenfunctions $\varphi_i(\mathbf{z})$ are retained in equation (10). These KL terms account for 90% of the variability of the Young's modulus random field. Thus, the uncertainty propagation problem is recast as a function of 24 independent standard normal variables $u_i(\omega), i \in \{1, \dots, 24\}$. Hence, for a given realization of these random variables, a realization $\hat{E}(\mathbf{z}, \omega)$ of the random field representing the Young's modulus of the cracked plate is obtained from equation (12). Figures 2 shows a sample of 10 realizations of $\hat{E}(\mathbf{z}, \omega)$.

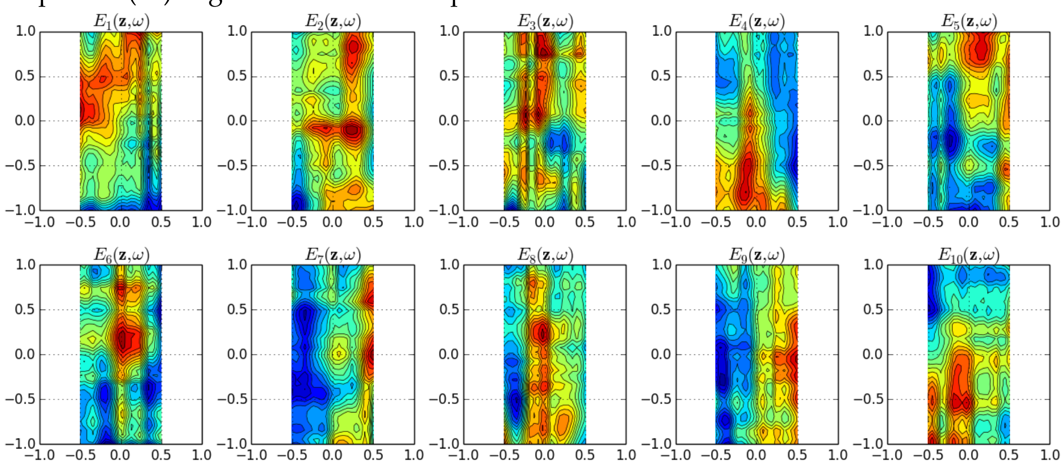

Figure 2 Inclined edge-cracked plate: example of 10 realizations of the Young's modulus field $E(\mathbf{z}, \omega)$ with mean $\mu_E = 20.7\ 10^6$ units, standard deviation $\sigma_E = 2.07\ 10^6$ units, horizontal correlation length $l_{cx} = 0.5$ unit and vertical correlation length $l_{cy} = 1.5$ units

As a first step, a statistical moments and distribution analysis is performed to assess the effect of the spatial randomness of the Young's modulus on the variability of the fracture driving forces. The statistical moments of each model response are computed by the full-PCE and sparse-PCE approaches. The results obtained for the first two first statistical moments, i.e., the mean and the standard deviation, with a PCE of degree $p = 2$, are listed in Table 1 and compared to the estimates given by efficient cubature (Chahine, 2023) and 10^5 crude MCS. As can be seen, the results given are in complete agreement. The discrepancy with respect to the reference estimates given by 10^5 MCS is insignificant for all the mechanical responses of interest. It appears that the uncertainty on the Young's modulus, i.e., 10% deviation from its mean value, has a moderate effect on the variability of the crack driving forces, since the coefficients of variation corresponding to the opening fracture mode SIF K_I, the in-plane shear fracture mode SIF K_{II}, the bifurcation angle θ and the effective SIF K_{eff}, are equal to 2.75%, 3.97%, 1.55% and 2.93%, respectively. The truncation of the polynomial chaos basis based on second moment information significantly reduces the computational effort devoted to solving the least-square regression problem used in the sparce-PCE approach to estimate the PCE coefficients. Indeed, only 25 of the 325 components of the full polynomial chaos basis $\mathcal{H}^p$ have significant contributions on the model responses. The corresponding economy index $\mathcal{E}^{p,q,\sigma^2} = 100 \times (\text{card}(\mathcal{H}^p) - \text{card}(\mathcal{H}^{p,q,\sigma^2})/\text{card}(\mathcal{H}^p))$ is about 92%, which shows high sparsity in the truncated polynomial chaos basis $\mathcal{H}^{p,q,\sigma^2}$.

Table 1 Inclined edge-cracked plate: statistical moments of the crack driving forces K_I, K_{II}, θ and K_{eff}

	Statistical moments	Full-PCE	Sparse-PCE	Crude cubature	MCS
K_I	μ	2.8253	2.8253	2.8253	2.8255
	σ	0.0779	0.0781	0.0781	0.0778
K_{II}	μ	1.2061	1.2061	1.2061	1.2061
	σ	0.0460	0.0460	0.0460	0.0479
θ	μ	36.776	36.776	36.776	36.774
	σ	0.5570	0.5570	0.5570	0.5713
K_{eff}	μ	6.8846	6.8846	6.8846	6.8849
	σ	0.1992	0.1992	0.1992	0.2022
	Number of FEM runs	651	651	651	10^5

Next, a sensitivity analysis is conducted to assess the contribution of the uncertain parameters $u_i(\omega), i \in \{1, \dots, 24\}$, resulting from the representation of the random field $E(\mathbf{z}, \omega)$ by a 24th order KL expansion, on the variability of the effective SIF K_{eff}. It is important to remind that this effective crack driving force, which is derived from the opening fracture mode SIF K_I, the in-plane shear fracture mode SIF K_{II} and the bifurcation angle θ, can be considered from a physical point of view as an opening fracture mode SIF in the direction along the bifurcation angle θ. This parameter is of a great importance when dealing with mixed-mode fracture problems since it is used in the computation of the fatigue crack growth life instead of K_I and K_{II}. Moreover, when a reliability analysis is to be performed with respect to a serviceability criterion function of the fracture toughness of the constitutive material, the effective SIF K_{eff} should also be used.

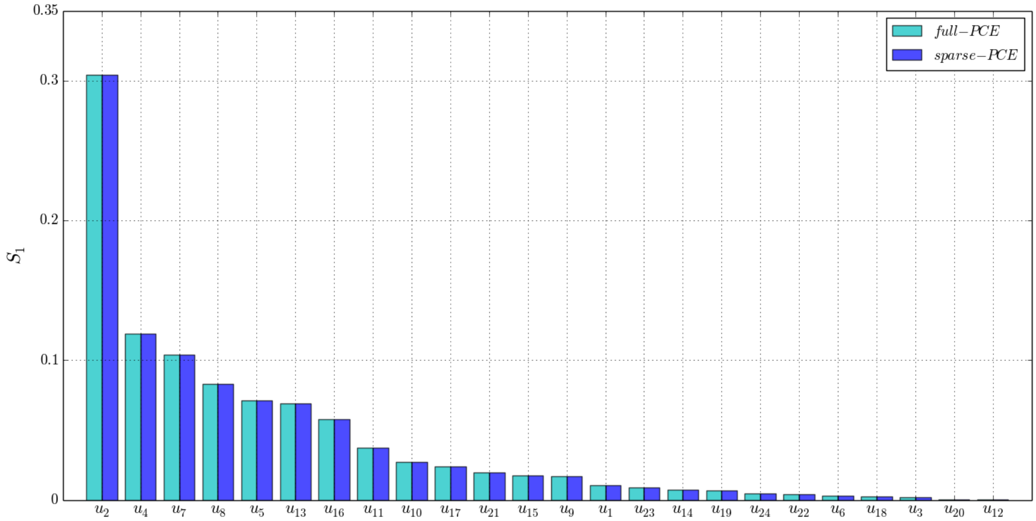

Figure 3 Inclined edge-cracked plate: comparison of the estimates of the first-order Sobol indices

Due to the high probabilistic dimension of the problem, the evaluation of Sobol indices by MCS impractical. Therefore, the following sensitivity analysis relies only on the full-PCE and sparse-PCE approaches. Figure 3 compares the estimates of the first-order Sobol indices obtained by post-processing the PCE coefficients of the metamodels given by the full-PCE and sparse-PCE approaches. As can be seen, the first-order sensitivity indices given by both the full-PCE and sparse-PCE approaches are practically identical. This fact can be considered as an indicator of convergence for the obtained estimates, and they can therefore represent the reference solution. A very fast decay of the main effect of the uncertain parameters is observed. Moreover, the uncertain parameter $u_2(\omega)$, corresponding to 2nd eigenmode of the KL expansion, is by far the most significant effect among all the uncertain parameters, whereas $u_{20}(\omega)$ and $u_{12}(\omega)$ have almost no effect on the variability of the effective SIF K_{eff}. In figure 6 are depicted the total Sobol indices. The estimates provided by the full-PCE and sparse-PCE approach are quite similar. We observe that the order of importance of the uncertain parameters is the same as for the first-order indices. The sum of the total indices $\sum_{i=1}^{24} S_{Ti}$ is approximately equal to 1, which means that the interactions between the uncertain parameters have weak effects on the model response of interest. Indeed, if we compare the total indices with the respective first-order ones, it appears that the differences are negligible, again demonstrating the insignificance of the contributions of the interaction effects. As pointed out in Figure 4, a very fast decay of the importance of the uncertain parameters is observed, with the 10 first uncertain parameters $u_i(\omega), i \in \{2, 4, 7, 8, 5, 13, 16, 11, 10, 17\}$ explaining roughly 90% of the total variance of the effective SIF K_{eff}. This demonstrates a moderate effective probabilistic dimensionality of the mechanical response of interest despite the large nominal probabilistic dimension, 24, corresponding to the number of eigenmodes required by the KL expansion to accurately represent the spatially varying uncertainty in the Young's modulus of the constitutive material of the cracked plate. Although we do not have a true reference solution for the Sobol sensitivity indices, the obtained estimates are in good agreement with the results of the local sensitivity analysis conducted by (Long and al, 2016), since it has been shown that the uncertain parameters $u_2(\omega)$ and $u_4(\omega)$, corresponding to the 2nd and 4th eigenmodes of the KL expansion, respectively, are the most important on the variability of the SIFs of K_I and K_{II}. Indeed, the local sensitivity indices obtained by the central difference method with respect to the uncertain parameters $u_2(\omega)$ and $u_4(\omega)$, are respectively $\partial K_I/\partial u_2 = -0.0292$ and $\partial K_I/\partial u_4 = 0.0270$ for the opening fracture mode SIF K_I, and $\partial K_{II}/\partial u_2 = -0.0254$ and $\partial K_{II}/\partial u_4 = 0.0105$ for the in-plane shear fracture mode SIF K_{II}.

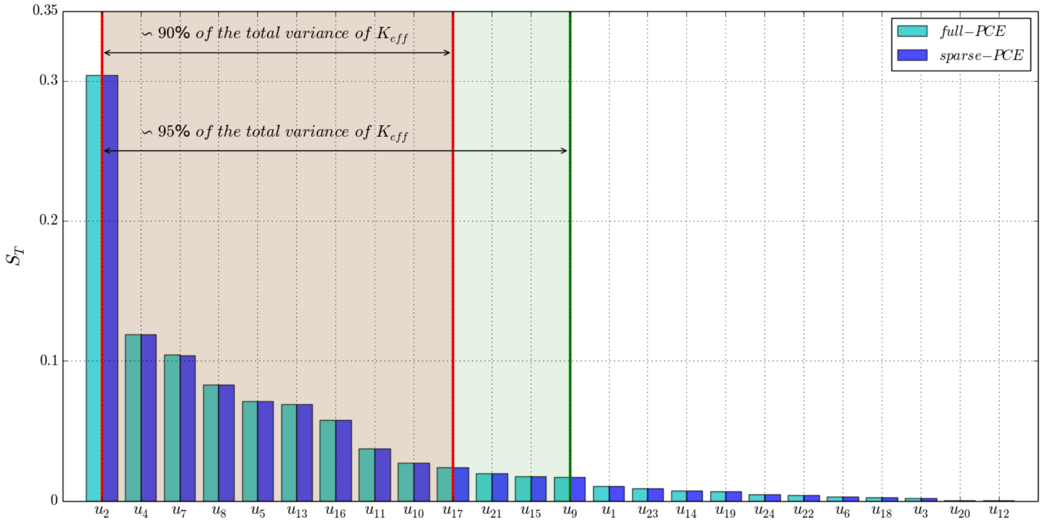

Figure 4 Inclined edge-cracked plate: comparison of the estimates of the total Sobol indices

It is important to notice that the sensitivity analysis conducted here did not require any additional runs of the FEM, since the Sobol sensitivity indices are derived from the coefficients of the metamodels already built in the statistical moments and distribution analysis conducted earlier.

IV. Conclusion

We demonstrate the efficiency of the sparse-PCE approaches for conducting different types of uncertainty propagation analysis through a computationally demanding implicit mechanical model. The proposed approach reduces the computational effort by at least a factor of two, and possibly a factor of three if a reliability analysis is carried out later. It is worth noting that the computational cost required by the sparse-PCE approach is due to the derivation of second moment information needed to build the sparse chaos polynomial basis, rather than the estimation of the PCE coefficients. If prior second moment information are already available, the computational cost gain should be more noticeable.

It is important to recall here that the idea behind the implementation of the sparse-PCE approach is to avoid the additional computational efforts observed when cubature formulae are directly used on the mechanical model, and when one wishes to change the type of uncertainty propagation analysis. For instance, a statistical moments analysis can be carried out, first, to provide a target estimate of the variance of the model response, and then the sparse-PCE approach is used to construct a metamodel that can be used to perform either a sensitivity or a reliability analysis. The accuracy of the sparse-PCE approach can be improved when the stopping criteria of the stepwise algorithm is established based on higher-order statistical moments such as skewness and kurtosis, instead of variance, provided that the mechanical model evaluations already available are sufficient to obtain a well-conditioned regression problem.

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