Reliable design optimization of a cantilever beam structure by using Dirlik fatigue approach

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ABSTRACT

Reliability-Based Design Optimization (RBDO) is a widely used approach for optimizing engineering structures, but it often neglects the effects of fatigue. In this article, we propose an RBDO methodology that combines the Reliability Index Approach (RIA) with Dirlik spectral method for fatigue damage assessment. The RIA approach is utilized to estimate the reliability index (RI) of the structure, while Dirlik fatigue approach is employed to assess the fatigue damage. Fatigue damage is introduced as a hard constraint in the optimization process. A case study is presented to demonstrate the effectiveness of the proposed methodology. The results show that the RBDO approach, incorporating fatigue phenomenon using RIA and Dirlik spectral method, can lead to reliable and fatigue-informed optimal designs for enhanced performance and durability of cantilever beam structures. Further research can explore the applicability of the proposed methodology to other types of structures and loading conditions.

Keywords Fatigue damage, Reliability based design optimization (RBDO), Random vibration, Dirlik approach

I. INTRODUCTION

In a competitive economic context, industrial sectors such as aeronautics or automotive are now subject to environmental regulations. To reduce greenhouse gas emissions, they have implemented solutions such as lightening structures by reducing their volume. This lightening requires new studies to ensure the product reliability during operating life. Thus, reliability-based design optimization (RBDO) methods can provide solutions with a good compromise between volume and safety, by minimizing structures cost under reliability constraints (Aoues and Chateauneuf, 2010). However, mechanical structures in their operating environments can become unsafe due to fatigue phenomenon. This phenomenon occurs when mechanical structures are subject to a cyclic load that isn't strong enough to cause failure if applied once, but when applied repeatedly, it can lead to the formation of a crack that spreads leading to failure.

Fatigue damage can affect many industrial fields in which mechanical structure are subject to random vibrations (wind, waves, etc.) in their operating conditions, leading to premature failure and compromising human and material safety. In fact, fatigue is the source of 55% of structural failures in aircraft (Wild et al., 2021), and 90% of all service failure in general (Fajri et al., 2021; F.C.

Campbell, 2016). It is therefore essential to consider this phenomenon in the structural design optimization process (Aoues et al., 2017; Lee et al., 2015).

One of the first studies that implemented fatigue parameter in optimization was Haiba et al. (Haiba et al., 2005) by introducing an evolutionary structural optimization that minimizes the structure by removing the finite element sets with the highest expected life. Pagnacco et al. (Pagnacco et al., 2012), developed a method based on the Sines' criterion to calculate fatigue damage which is introduced as a constraint in the optimization problem formulation. One of the first studies to consider the fatigue in Reliability Based Design Optimization (RBDO) is by Yang and Wang (Yang and Wang, 2012) where they used metamodels and calculated the damage from ε-N curves and Miner's law in the time domain. Lambert et al. (Lambert, 2007) used Evolutionary Structural Optimisation (ESO) algorithm, with a sensitivity damage criterion evaluated in the optimization algorithm. A study by Aoues et al. (Aoues et al., 2017) used the Sines' criterion for damage calculation in the frequency domain and introducing it into RBDO. In topology optimization, (Lee et al., 2015) considered the lifetime in the frequency domain by estimating the damage by spectral methods. While Oest and Lund (Oest and Lund, 2017) did the topology optimization by introducing the fatigue phenomenon by damage calculation in the time domain with S-N curves, Rainflow counting and Miner's law, but the reliability was not considered as an optimization criterion.

This work introduces a method that integrates fatigue damage constraints, in the Reliability Index Approach (RIA) of RBDO while considering uncertainties in loading and inputs parameters. The fatigue damage is assessed in the frequency domain using spectral methods. In order to focus our presentation on methodological aspects, the chosen application is a cantilever beam subjected to a Gaussian stationary ergodic random loading. This allows us to highlight the key methods and techniques employed, rather than getting overly involved in the particulars of the application scenario.

The first part of this work is dedicated to the introduction of RBDO, and fatigue damage assessment methods. Then, an analytical-numerical application to a cantilever beam is presented in the second part.

II. Fatigue reliability-based design optimization of a cantilever beam

RBDO approaches in structural mechanics, aim to find a good compromise between cost and safety (Aoues and Chateauneuf, 2010), by minimizing structures volumes while respecting a reliability threshold. In this study we propose to optimize the design of *a cantilever* beam to reduce its cost while respecting a reliability threshold and considering fatigue phenomenon. This steel beam is subjected at its fixed *end* to *a* random vibration defined by an acceleration spectral density (*A*SD). Random vibration analysis is used to estimate the damage to the beam, taking into account a load close to what it might experience during its lifetime. Random vibration analysis is a method used to study the response of a structure subjected to a random vibration input. After a non-exhaustive presentation of the RBDO, modal analysis of a cantilever beam is presented. Subsequently, this will allow us to assess fatigue life using spectral methods that will be introduced as fatigue constraints in the RBDO formulation.

A. Reliability based design optimization

Optimization problems in RBDO are formulated as:

$$\min_{d} C_I(\boldsymbol{d}) \tag{1}$$

$$sc. \quad \begin{cases} P_f(\boldsymbol{d},\boldsymbol{X}) = Prob[G_i(\boldsymbol{d},\boldsymbol{X}) \geq 0] \leq P_f^t \\ h_j(\boldsymbol{d}) \geq 0 \end{cases} \tag{2}$$

Where $\boldsymbol{X}$ is the random variable vector representing the Young's modulus and the beam density, $\boldsymbol{d}$ is the design variables vector representing the length and height of the beam, h_j is the j-th deterministic constraint that represent the intervals of the design variables, C_I is the initial cost, G is the limit state function, P_f is the corresponding failure probability, and P_f^t is the target failure probability. It should be noted that the limit state G corresponds to the difference between the estimation and the threshold of fatigue.

This formulation is at the origin of all RBDO methods, and it can be solved with gradient-based or non-gradient-based optimization algorithms. RBDO methods can be classified into 3 categories: Double-Level Approach (DLA), Single-Level Approach (MLA) and Sequential Decoupled Approach (SDA), further details on these approaches can be found in (Aoues and Chateauneuf, 2010).

In this work we will use the Reliability index approach RIA which is a double level approach for its simplicity in implementation (Aoues and Chateauneuf, 2010) . This approach was introduced by Enevoldsen and Sorensen (Enevoldsen and Sorensen, 1994) and it involves formulating the reliability constraints in terms of a target reliability index value, rather than specifying an upper bound on the probability of failure. The principle of this approach is to replace the probability of failure constraints by constraints of the reliability index β, based on the First Order Reliability Method (FORM), the formulation of this approach is (Aoues and Chateauneuf, 2010):

$$\min_{d} C_I(\boldsymbol{d}) \tag{3}$$

$$sc. \quad \begin{cases} \beta(\boldsymbol{d},\boldsymbol{X}) \geq \beta^t \\ h_j(\boldsymbol{d}) \geq 0 \end{cases} \tag{4}$$

Where β is the reliability index of limit state G, β^t is the reliability index target which is given by FORM as $\beta^t = -\phi^{-1}(P_f^t)$ where ϕ is the cumulative function of normal distribution. The reliability index corresponds to the minimum distance between the origin of the normalized space of optimization variables and the limit state. It can be obtained by formulating a constrained optimization problem, which can be solved with optimization algorithms:

$$\beta = \min_{d} ||u|| \tag{5}$$

$$sc. \quad \hat{G}(\boldsymbol{U}) \leq 0 \tag{6}$$

where $\hat{G}(U)$ is the limit state function of the normalized space, and $\boldsymbol{U}$ is the vector of centered and decorrelated normalized random variables. This reliability index optimization problem is usually solved with Hasofer Lind- Rackwitz Fiessler (HL-RF) algorithm (Rackwitz and Fiessler, 1978) whose concept is to find iteratively in a descent direction $\alpha^{(k)}$, the most likely point of failure, then, the new point is defined with the following equation:

$$u^{(k+1)} = -\beta^{(k)}\alpha^{(k)} \tag{7}$$

with
$$\beta^{(k)} = u^{(k)}\alpha^{(k)} + \frac{G(u^{(k)})}{\|\nabla G(u^{(k)})\|} \tag{8}$$
and
$$\alpha^{(k)} = \frac{\nabla G(u^{(k)})}{\|\nabla G(u^{(k)})\|} \tag{9}$$

The RIA double loop approach involves iterating between these two loops until the desired reliability level is achieved. The outer loop updates the design variables to minimize the cost while meeting the reliability constraints and the inner loop calculates the reliability index using the current design variables. (Aoues and Chateauneuf, 2010; Oest and Lund, 2017).

C. Random vibration

Our system consists of a cantilever beam attached to the fixture of a shaker that emits random vibrations, resulting in a motion of the system $u(x,t)$. The motion consists of the combined movements of the armature $w(x,t)$ and the specimen $y(x,t)$. This the general equation of this system is (S. S. Rao, 1990) :

$$\rho\frac{\partial^2 u}{\partial t^2} + C\frac{\partial u}{\partial t} - \frac{\partial}{\partial x}\left[T\frac{\partial u}{\partial x}\right] + \frac{\partial^2}{\partial x^2}\left[EI\frac{\partial^2 u}{\partial x^2}\right] = f(x,t) \tag{10}$$

This equation presents an equilibrium equation of force per unit length that defines the response of the structure at each point and each moment. This equilibrium is composed of inertia, damping, and stiffness, which in turn is composed of a term for axial tension force T and the bending stiffness EI(McConnell and Varoto, 1995). The absolute motion of the specimen $u(x,t)$ is related to the motion of the test fixture $w(x,t)$and the relative motion of the specimen $y(x,t)$ by:

$$u(x,t) = w(x,t) + y(x,t) \tag{11}$$

Assuming that the external excitation force is zero, and that the major contribution of the damping comes from the internal energy dissipation mechanisms, and that $w(x,t)$ is linear to x (motion of a rigid body) the differential equation of the system motion is:

$$\rho\frac{\partial^2 y}{\partial t^2} + C\frac{\partial y}{\partial t} + \frac{\partial^2}{\partial x^2}\left[EI\frac{\partial^2 y}{\partial x^2}\right] = -\rho\frac{\partial^2 w}{\partial t^2} \tag{12}$$

with ρ the beam density, C the damping, E Young modulus, I moment of inertia, $y(x,t)$ the beam displacement, and $\frac{\partial^2 w}{\partial t^2}$ the excitation. By assuming an undamped structure without excitation, Equation (1) becomes:

$$EI\frac{d^4 y}{dx^4} = -\rho\frac{d^2 y}{dt^2} \tag{13}$$

By considering only the spatial part of the displacement, and using boundary conditions, the mass normalized eigenfunctions are:

$$Y_n(x) = \left\{\frac{1}{\sqrt{\rho L}}\right\}\{[\cosh(\eta_n x) - \cos(\eta_n x)] - D_n[\sinh(\eta_n x) - \sin(\eta_n x)]\} \tag{14}$$

with
$$D_n = \frac{\cosh(\eta_n L) + \cos(\eta_n L)}{\sinh(\eta_n L) + \sin(\eta_n L)} \tag{15}$$

$Y_n(x)$ is the displacement space component, η_n the eigenvalues, and L the beam length defined for the *n-th* mode.

The structure displacement is represented as a sum of modal displacements, where each mode represents a particular shape of vibration. The solution to equation (12) can be separated into two components: one that depends on time and another that depends on space (Irvine, 2013; Thomson, 1993):

$$y(x,t) = \sum_{n=1}^{m} Y_n(x).T_n(t) \tag{16}$$

If we replace now in equation (12), and by using orthogonality condition of Sturm Liouville (Farzana et al., 2015):

$$\left(Y_n, Y_p\right) = \int_0^L Y_n(x)Y_p(x)\,dx = \delta_{mn} = \begin{cases} 0 \; for \; n \neq p \\ 1 \; for \; n = p \end{cases} \tag{17}$$

We can obtain the following equation:

$$\frac{d^2}{dt^2}T_n(t) + \omega_n^2 T_n(t) = -\Lambda_n \frac{d^2 w}{dt^2} \tag{18}$$

With ω_n the eigen frequency in (rad/s) and Λ_n the modal participation factor defined as:

$$\Lambda_n = \int_0^L \rho Y_p(x)dx \tag{19}$$

With Y_n, Y_p two different mode shapes, T_n the time component in the displacement.

The modal damping given in equation (12) is defined as:

$$\frac{C}{\rho} = 2\xi_n\omega_n \tag{20}$$

Thus, equation (18) becomes:

$$\ddot{T}_n(t) + 2\xi_n\omega_n\dot{T}_n(t) + \omega_n^2 T_n(t) = -\Lambda_n\ddot{w}(t) \tag{21}$$

To convert the equation from time domain to frequency domain this equation, we apply the Fourier transform (FT[■]):

$$\mathrm{FT}[\ddot{T}_n(t) + 2\xi_n\omega_n\dot{T}_n(t) + \omega_n^2 T_n(t)] = \mathrm{FT}[-\Lambda_n\ddot{w}(t)] \tag{22}$$

leading to:

$$\left[(\omega_n^2 - \omega^2) + 2j\xi_n\omega_n\omega\right]\int_{-\infty}^{+\infty} T_n(t)e^{-j\omega t}dt = -\Lambda_n\int_{-\infty}^{+\infty}\ddot{w}(t)e^{-j\omega t}dt \tag{23}$$

We can write equation (23) as:

$$Z_n(\omega) = \frac{-1}{[(\omega_n^2 - \omega^2) + 2j\xi_n\omega_n\omega]}\Lambda_n\ddot{W}(\omega) \tag{24}$$

with

$$Z_n(\omega) = \int_{-\infty}^{+\infty} T_n(t)e^{-j\omega t}dt \tag{25}$$

and

$$\ddot{W}(\omega) = \int_{-\infty}^{+\infty}\ddot{w}(t)e^{-j\omega t}dt \tag{26}$$

Equation (24) represents the frequency component of the displacement. Furthermore, the relationship between the input excitation $\ddot{W}(\omega)$ and the displacement response $Z_n(\omega)$ is defined as the Frequency Response Function (FRF) of n-th mode, expressed by the ratio between output and input.

$$H_n(\omega) = \frac{\ddot{W}(\omega)}{Z_n(\omega)} = \frac{-\Lambda_n}{[(\omega_n^2 - \omega^2) + 2j\xi_n\omega_n\omega]} \tag{27}$$

With equation (16) and (24) We get the general expression of the displacement:

$$y_n(x,\omega) = \ddot{W}(\omega)\sum_{n=1}^{m}\left\{\frac{-\Lambda_n Y_n(x)}{[(\omega_n^2 - \omega^2) + 2j\xi_n\omega_n\omega]}\right\} \tag{28}$$

And the stress can be defined as:

$$\sigma_n(x,\omega) = \frac{h}{2}E\frac{\partial^2}{\partial x^2}y_n(x,\omega) = \frac{h}{2}E\sum_{n=1}^{m}\left\{\frac{-\Lambda_n \frac{d^2}{dx^2}Y_n(x)}{[(\omega_n^2 - \omega^2) + 2j\xi_n\omega_n\omega]}\right\}\ddot{W}(\omega) \tag{29}$$

The PSD (Power Spectral Density) is a statistical measure of the energy content of a vibration signal as a function of frequency. In random vibration analysis for fatigue, the PSD is used to characterize the input vibration signal $X(t)$. The PSD is a critical parameter in fatigue analysis because the amount of energy contained in the vibration signal can significantly impact the fatigue life of a structure or component. It is defined as the Fourier Transform (FT) of the autocorrelation function as follows:

$$S_{XX}(\omega) = \mathrm{FT}[R_{XX}(\tau)] = \int_{-\infty}^{+\infty} R_{XX}(\tau)\exp(-j\omega\tau)\,d\tau \tag{30}$$

with $S_{XX}(\omega)$ the PSD function, and $R_{XX}(\tau)$ the autocorrelation function. For a stationary and Gaussian process, the response PSD can be expressed as:

$$S_{YY}(\omega) = |H(\omega)|^2 S_{XX}(\omega) \tag{31}$$

where: $H(\omega)$ is the FRF and $S_{YY}(\omega)$ is the response PSD.

From equations (27), (29) and (31), we can obtain the stress response PSD equation defined as:

$$S_{\sigma_n\sigma_n}(\omega) = \left|\frac{h}{2}E\sum_{n=1}^{m}\left\{\frac{-\Lambda_n \frac{d^2}{dx^2}Y_n(x)}{[(\omega_n^2 - \omega^2) + 2j\xi_n\omega_n\omega}\right\}\right|^2 S_{\ddot{W}\ddot{W}}(\omega) \tag{32}$$

With $S_{\ddot{W}\ddot{W}}$ the input acceleration PSD, and h is the beam height.

D. Fatigue life assessment

Fatigue is a phenomenon in mechanical structures where a material undergoes progressive and localized damage due to repeated cyclic loading. The fatigue process can result in a reduction in the structural integrity of a component or structure, leading to failure over time.

One of the methods for fatigue damage assessment is done in the frequency domain through spectral methods. The Miner's damage can be expressed as an expected value given by:

$$\mathrm{E}[D] = C^{-1}\nu_0\int_0^{\infty}\sigma_a{}^b p_s(\sigma_a)d\sigma_a \tag{33}$$

where C and b are material parameters, σ_a is the amplitude of applied stress, $p_s(\sigma_a)$ is the stress probability density function and ν_0 is the rate of zero up crossings defined in equation (37). The probability density function can be approximated by spectral methods. In this study, we used Dirlik method for fatigue estimation because of its efficiency (Dirlik, 1985; Mršnik et al., 2013;

Demirel and Kayran, 2019). Dirlik defines the stress probability density function as a combination of an exponential distribution and two Rayleigh distributions, it's given by:

$$\mathrm{E}[D^{Dir}] = C^{-1}\nu_p T\left(\sqrt{m_0}\right)^b \left[D_1 Q^b \Gamma(1+b) + \left(\left(\sqrt{2}\right)^b\right)\Gamma\left(1+\frac{b}{2}\right)(D_2|R|^b + D_3)\right] \tag{34}$$

With Γ[■] the gamma function and the parameters introduced by Dirlik were defined as:

$$\begin{cases} Z = \dfrac{\sigma_a}{\sqrt{m_0}} \\ D_1 = \dfrac{2(x_m-\gamma^2)}{1+\gamma^2} \\ D_2 = \dfrac{1-\gamma-D_1+D_1^2}{1-R} \\ D_3 = 1-D_1-D_2 \end{cases} \quad \text{and} \quad \begin{cases} x_m = \dfrac{m_1}{m_0}\sqrt{\dfrac{m_2}{m_4}} \\ \gamma = \dfrac{m_2}{\sqrt{m_0 m_4}} \\ R = \dfrac{\gamma - x_m - D_1^2}{1-\gamma-D_1+D_1^2} \\ Q = \dfrac{1.25(\gamma-D_3-RD_2)}{D_1} \end{cases} \tag{35}$$

where m_1, m_2 and m_4 are the spectral moments defined as:

$$m_k = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |\omega|^k S_{XX}(\omega)\mathrm{d}\omega \tag{36}$$

ν_0, ν_p and γ are the rate of zero up crossings, the rate of peaks and the irregularity factor, respectively. ν_p is given by:

$$\nu_0 = \sqrt{\frac{m_2}{m_0}}\ ;\ \nu_p = \sqrt{\frac{m_4}{m_2}}\ ; \tag{37}$$

III. Analytical-numerical application

To test our method, a Steel cantilever beam of 1m length (FIGURE 2), is subjected to a base acceleration PSD as shown in FIGURE 1. The random variables statistical parameters are represented in TABLE 1.

TABLE 1. statistical parameters of random variables

Random variable	Mean	Standard deviation	Distribution
Young modulus E (MPa)	210000	2100	Normal
Density ρ (Kg/m³)	7900	79	Normal

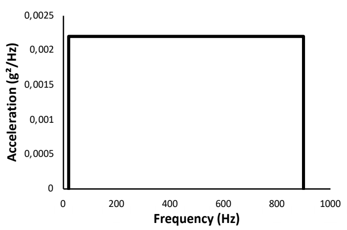

FIGURE 1. Input PSD acceleration

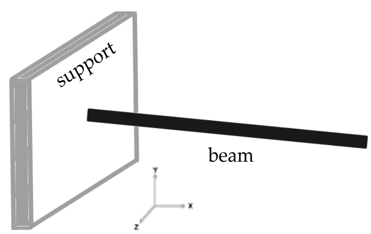

FIGURE 2. Cantilever beam

To validate the model for damage assessment, the numerical and analytical results were compared, then the computed damage was introduced into the reliability-based structural optimization problem as the hard constraint. The lower and upper bounds of the optimization variables are chosen so that the first and second eigenmodes are within the excitation PSD frequency bandwidth, as these modes are the most damaging. The cross section of the beam (height and width) is minimized to reduce its cost, under the constraint that the beam fatigue life should be greater than or equal to a fixed year (i.e., 2, 5, 10, 20, … years). The optimization problem is written:

$$\min_{d} C_I(d) = H * W$$
$$sc. \begin{cases} \beta \geq 2.9 \\ 20 \leq f_1 \leq 900 \\ 20 \leq f_2 \leq 900 \\ W \geq H \\ 0.03 \leq H \leq 0.3 \\ 0.03 \leq W \leq 0.3 \end{cases} \tag{38}$$

With f_1 and f_2 the eigen frequencies of the two first modes, H and W the height and width of the beam respectively. As mentioned before the reliability index is obtained by minimizing the distance between the origin of the normalized space of optimization variables and the limit state. The limit state in our study is:

$$G = \frac{T_{fatigue}}{T_{target}} - 1 \ ; \tag{39}$$

With $T_{fatigue}$ the fatigue life calculated with Dirlik method, and T_{target} the life service target of 2 years. The results of the optimization shown in TABLE 2 show that the method leads to an optimal design, while keeping safety conditions respected and having a reliability index of 2.9 corresponding to a failure probability of 0.0019.

TABLE 2. Results of RBDO with a life service target of 2 years

	Initial design	Optimized design
Height (mm)	100	44.298
Width (mm)	100	46.77
Cross section (mm²)	10000	1983.221
Fatigue life (years)	46	2
Failure probability	0.0019	0.0019

In order to observe how the height and width can evolve as a function of the required life service and the RI, an optimization was made with different RI and fatigue life targets, the results are shown in FIGURES 3, 4.

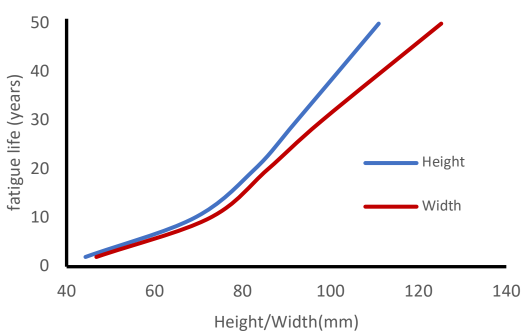

FIGURE 3. Fatigue life evolution in terms of Height and width

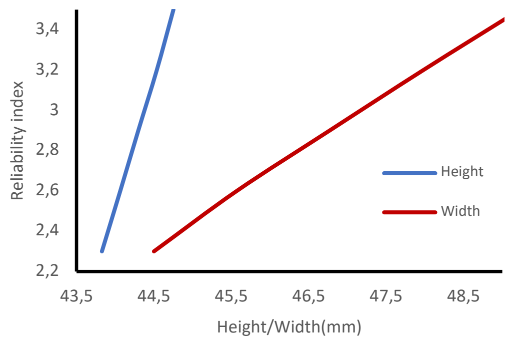

FIGURE 4. Reliability index evolution in terms of Height and width

The results in FIGURE 3 and 4, show that compared to the height of the beam, the width appears to have a smaller influence on the variation of fatigue life and RI especially for higher values of the latter. Furthermore, the fatigue life appears to vary significantly with even minor changes in the beam height and width. This implies that small alterations in the geometry of the beam can lead to significant variations in damage and ultimately affect the fatigue life of the beam. This underscores the importance of accurately considering the geometric parameters of a beam during the design and maintenance process to ensure its structural integrity and durability. Additionally, the reliability index, which provides a measure of the safety margin of the beam against failure, also exhibits sensitivity to changes in beam geometry. The different values of the reliability index in FIGURE 3, demonstrate that as the reliability index increases, indicating a higher safety margin, the width and height of the beam also increase which tends to improve the fatigue life. This suggests that higher reliability index values correspond to longer fatigue life and increased structural safety. The fluctuations in the values corresponding to 10 years fatigue life, may correspond to the uncertainties in the input parameters, as very small variations in the geometry can influence greatly the fatigue damage and therefore the fatigue life.

Overall, the findings from the curves analysis highlight the critical role of beam geometry, particularly the height, in determining the fatigue life and reliability of the beam. These results contribute to the understanding of the complex relationship between beam geometry, reliability index, fatigue life, and structural safety, and underscore the importance of accurate modeling and analysis in structural engineering practices.

IV. Conclusion

The proposed approach provides different reliable designs of the cantilever beam regarding several target lifetimes under random vibrations. Moreover, the reliability-based design optimization under fatigue damage limit state reduces the structural cost while preserving its safety requirements. The method used in this study can allow an economic geometrical choice that respects reliability and safety requirements.

The reliability optimization process involves multiple calls to the mechanical model (an average of 8000). Nevertheless, the computational time is relatively short due to the simplicity of the structure. However, for more complex structures, the computational efficiency may be significantly affected. Therefore, this work can be expanded to complex structures by using surrogate models in the reliability-based design optimization process allowing to reduce the computation time.

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