Simplified 1D model of groundwater transient flow through a multilayered riverbank soil.

Sambath KY[1], Juan MARTINEZ[2], Soksan CHHUN[3]

[1] *Institut National des Sciences Appliquées de Rennes, Laboratoire GCGM, 20, avenue des Buttes de Coësmes – CS 70839 – 35708 RENNES, France, email : sambath.ky@insa-rennes.fr*

[2] *Institut National des Sciences Appliquées de Rennes, Laboratoire GCGM, email : Juan.Martinez@insa-rennes.fr*

[3]*Institut de Technologie du Cambodge, Départment GCI, Russian Conf. Blvd, Phnom Penh, Cambodge, email : soksanchhun@yahoo.com*

RÉSUMÉ. Le niveau de la nappe phréatique le long des berges d'une rivière varie en fonction de la fluctuation du niveau d'eau de la rivière. Par conséquent, la stabilité des berges est sensible au niveau variable des pressions interstitielles générées. C'est pourquoi, pour évaluer le risque d'effondrement des berges, l'étude de l'écoulement transitoire de l'eau dans la berge, généralement multicouche, est de première importance. Le modèle d'écoulement simplifié 1D proposé est basé sur, (i) l'hypothèse de Dupuit de quasi-verticalité des lignes équipotentielles, (ii) l'équation de Boussinesq pour les écoulements non-permanents, et (iii) des perméabilités équivalentes et des porosités efficaces locales des couches de sol. Une discrétisation en différences finies dans l'espace et le temps du problème non linéaire de l'écoulement transitoire dans un sol multicouche est déduite. La résolution numérique est conduite dans le langage Matlab avec un algorithme itératif de Newton-Raphson. Les résultats montrent que le modèle simplifié proposé traduit bien le caractère multicouche du sol et l'influence de la perméabilité des différentes couches.

ABSTRACT. The groundwater table along the riverbanks varies as a function of the fluctuation of the river water level (WL). Consequently the riverbanks stability is sensitive to the generated pore water pressures. That is why to evaluate the risk of bank mass failure, the analysis of the transient water flow inside the bank, generally multilayered, is of prime importance. The proposed 1D simplified flow model is based on, (i) the Dupuit assumption of quasi-verticality of the equipotential lines (ii) the Boussinesq's equation for groundwater non-permanent seepage and (iii) local equivalent hydraulic conductivities and effective specific yields of the soil layers. A finite difference discretization in space and time of the nonlinear problem of the transient seepage in a multilayered soil is derived. The numerical resolution is lead in Matlab language with a Newton-Raphson iterative algorithm. The results show that the proposed simplified model expresses well the multilayered nature of the soil and the influence of the hydraulic conductivity of the different layers.

MOTS-CLÉS : pression interstitielle, différences finies, surface libre, modèle de Dupuit, Newton-Raphson.

KEY WORDS: pore water pressure, finite difference, groundwater table, Dupuit model, Newton-Raphson.

1. Introduction

Most rivers are subjected to fluctuations of the water level due either to seasonal climate changes or because of sudden rises produced for instance by a rainy storm or by the emptying of a dam reservoir. These fluctuations of the water level (WL) produce variations of the hydraulic forces acting on the riverbank, not only on its surface but also inside the bank due to the permeability of the soil [REZ 94, CHH 15]. This is the case of the Lower Mekong, which is submitted to annually WL fluctuations due to the Monsoon regime, which discharge ratio may reach up to twenty times with fluctuations amplitude of the WL higher than 15 m [MRC 05, MIN 16]. These fluctuations vary the groundwater table in the bank, and the corresponding pore water pressures. These water pressure variations affect the mechanical equilibrium of the bank [REZ 94, CHH 13, OYA 15] and are admitted as an important factor triggering bank's mass failures [RIN 13]. The flow regime inside the bank can be modelled using general Darcy transient flow model. In this paper we present a simplified numerical solution of horizontal transient seepage applied to a multilayered soil as part of a simplified integrated numerical tool for the evaluation of riverbank mass failures.

2. Unconfined groundwater flow modeling

The problems of unsteady groundwater flow, involving a changing groundwater table can be obtained from the Dupuit-Forchheimer [FIT 02] approximation of quasi-verticality of the equipotential lines (Figure 1) and considering Darcy's law, mass balance and that the water and the soil are incompressible. The 1-D Boussinesq ground water equation with a local rectangular Cartesian coordinate system can be derived as [1]:

$$\frac{\partial}{\partial x}(K^x \cdot H \cdot \frac{\partial H}{\partial x}) = S^y \cdot \frac{\partial H}{\partial t} \qquad [1]$$

where: K^x [m/s] is the saturated hydraulic conductivity in direction x; H [m] is the height of water table (Figure 1); S^y is the specific yield (or drainage porosity); *t and x* are the time and space coordinates respectively.

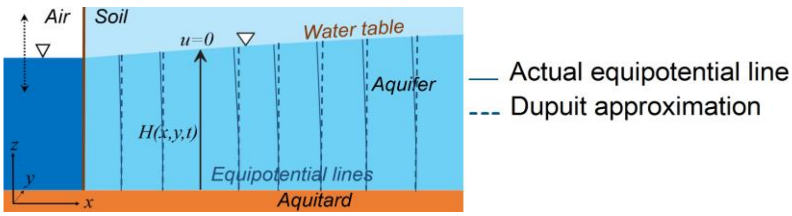

Figure 1. *Vertical cross-section of unconfined flow after Dupuit approximation*

2.1 Equivalent hydraulic conductivity of a multilayered soil

In a multilayered soil, each layer l has its own hydraulic conductivity K^{xl}. An equivalent hydraulic conductivity value of K^{xe} can be found by replacing at each abscissa x the different layers under the water table by an equivalent homogeneous vertical section. By supposing that the hydraulic gradient in the x direction in all soil layers is the same (vertical equipotential lines), the accumulative discharge ΣQ_{xl} of the set of layers is equal to the equivalent homogeneous one: $Q_{ex} = \Sigma Q_{xl}$ Applying Darcy's law to each layer and to the equivalent medium, the equivalent hydraulic conductivity can be defined as [2] [FIT 02]:

$$K^{xe} = \frac{\sum K^{xl} d^l}{\sum d^l} \qquad [2]$$

where d^l is the thickness of layer l under the water table.

As the layers thickness and the water table may vary with the abscissa x, the equivalent hydraulic conductivity K^{xe} is also locally dependent in space and time: $K^{xe} = K^{xe}(x,t)$.

2.2 Choice of the specific yield S^y

The specific yield S^y is related to the storage capacity of the soil during the groundwater table fluctuations. In a multilayered soil the layer that can store or release the water is the one containing the groundwater table. Thus the specific yield must be chosen as this of that layer. However, as the groundwater table position depends both on x and t, the specific yield is also a function $S^y(x,t)$ locally dependent in space and time.

3. Numerical solution

Discretization of equation [1] by finite difference using backward difference in time and a second-order central difference in space with local equivalent hydraulic conductivity and specific yield, gives the nonlinear equation [3]:

$$\frac{\Delta t}{2\Delta x^2}\left(\frac{K^e_{i+1/2,j+1}}{S^y_{i+1/2,j+1}}\cdot H^2_{i+1,j+1}-\left(\frac{K^e_{i+1/2,j+1}}{S^y_{i+1/2,j+1}}+\frac{K^e_{i-1/2,j+1}}{S^y_{i-1/2,j+1}}\right)\cdot H^2_{i,j+1}+\frac{K^e_{i-1/2,j+1}}{S^y_{i-1/2,j+1}}\cdot H^2_{i-1,j+1}\right)-H_{i,j+1}+H_{i,j}=0 \qquad [3]$$

where: i and j *are* the space and time index respectively; n is the total space index ($i = 1 \rightarrow n$); Δt and Δx are the time and space increment respectively; $H_{i,j}$ is the height of groundwater table at abscissa x_i, time t_j (Figure 2) . $K^e_{i+1/2,j+1}$ or $K^e_{i-1/2,j+1}$ and $S^y_{i+1/2,j+1}$ or $S^y_{i-1/2,j+1}$ are the equivalent hydraulic conductivities and the specific yields at space $x_i+\Delta x/2$ or $x_i-\Delta x/2$ respectively at time $t_j+\Delta t$; $K^e_{i+1/2,j+1}$ or $K^e_{i-1/2,j+1}$ is calculated by relation [2]. Solving equation [3] consists in finding the groundwater table $H_{i,j+1}$ at time $t_j+\Delta t$, knowing its position $H_{i,j}$ at time t_j, which requires to impose an initial condition $H_{i,0}$ and boundary conditions (Figure 2) $H_{1,j+1} = H_o(t_{j+1})$ at the riverside and $H(x_n,t_{j+1}) = H(x_{n+1},t_{j+1})$ far from the riverside (no input or output flow).

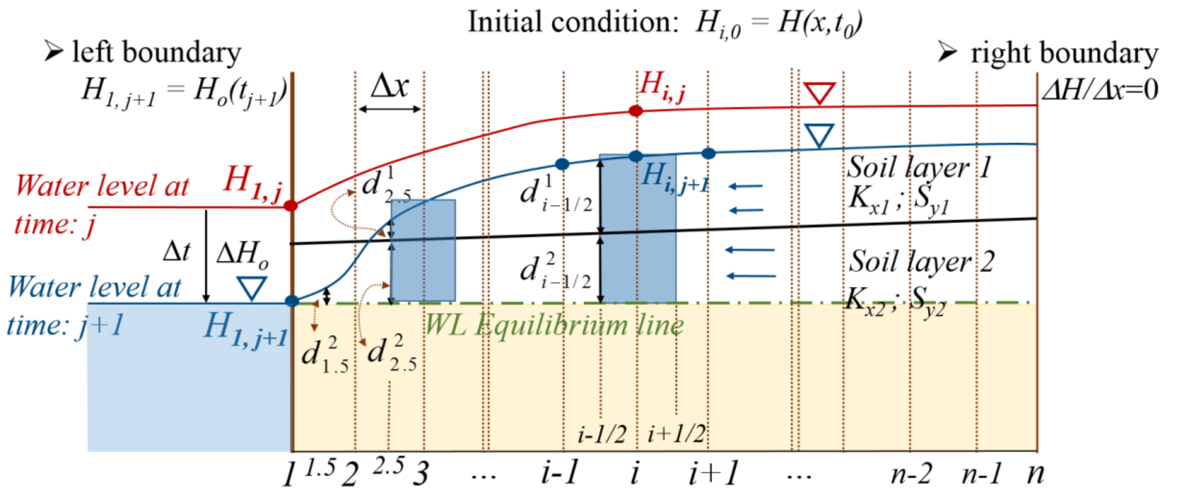

Figure 2. *Discretization of space and time for groundwater seepage model, (below the equilibrium line WL, the groundwater flow is assumed to negligible).*

A matrix formulation [4] is derived with n-1 equations and n-1 unknowns at each time step and an iterative Newton-Raphson algorithm is employed for treatment of the nonlinearity. The system is solved by using Matlab code.

$$\frac{\Delta t}{2\Delta x^2}[KS]_{n\times n,j+1}\times\left\{H^2\right\}_{n,j+1}-\left\{H\right\}_{n,j+1}+\left\{H\right\}_{n,j}=0 \qquad [4]$$

where:

$$[KS]=\begin{bmatrix}
1 & 0 & 0 & 0 & \cdot\cdot & 0 & 0 & 0 & 0\\
KS_{1.5} & -(KS_{1.5}+KS_{2.5}) & KS_{2.5} & 0 & \cdot\cdot & 0 & 0 & 0 & 0\\
0 & KS_{2.5} & -(KS_{2.5}+KS_{3.5}) & KS_{3.5} & \cdot\cdot & 0 & 0 & 0 & 0\\
0 & 0 & KS_{3.5} & -(KS_{3.5}+KS_{4.5}) & \cdot\cdot & 0 & 0 & 0 & 0\\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots\\
0 & 0 & 0 & 0 & \cdot\cdot & -(KS_{n-5/2}+KS_{n-3/2}) & KS_{n-3/2} & 0 & 0\\
0 & 0 & 0 & 0 & \cdot\cdot & KS_{n-5/2} & -(KS_{n-5/2}+KS_{n-3/2}) & KS_{n-3/2} & 0\\
0 & 0 & 0 & 0 & \cdot\cdot & 0 & KS_{n-3/2} & -(KS_{n-3/2}+KS_{n-1/2}) & KS_{n-1/2}\\
0 & 0 & 0 & 0 & \cdot\cdot & 0 & 0 & KS_{n-1/2} & -(KS_{n-1/2})
\end{bmatrix}$$

with $KS=\dfrac{K^e}{S^y}$;

$\{H\}_{n,j+1}$ is a vector $\{H_1 \rightarrow H_n\}_{j+1}$ with $H_{1,j+1}$ is the water level at time $j+1$;

4. Example of results

In the example below, we consider a vertical bank with three different soil layers and typical values of the hydraulic conductivities and specific yields. A WL linear drawdown is supposed from 42 m to 32 m in 50 days, representative of variations of the groundwater table in Mekong River between rainy and dry seasons. The space and time increments are taken equal to *0.1 m* and *0.1 day* respectively. The results of the groundwater position at times of 0 (horizontal initial condition), 14, 30 and 50 days are shown in Figure 3. Examples (a) and (b) have the same geometry and WL drawdown condition, but the first and second layer soil parameters are reversed.

Figure 3 shows that the groundwater table position condition is influenced by the stratigraphy of the soil. In case *(a)* we observe a quite high level of the groundwater table when it is entirely situated in the clay layer *(t = 14 days)*, but it decreases rapidly when reaching the sand layer *(t = 30 days)* that acts as a drain. Compared to case *(a)*, we observe that in case *(b)* the groundwater table is lower at *t = 14 days* and higher at *t = 30 days*, showing again the drain function of the sand layer.

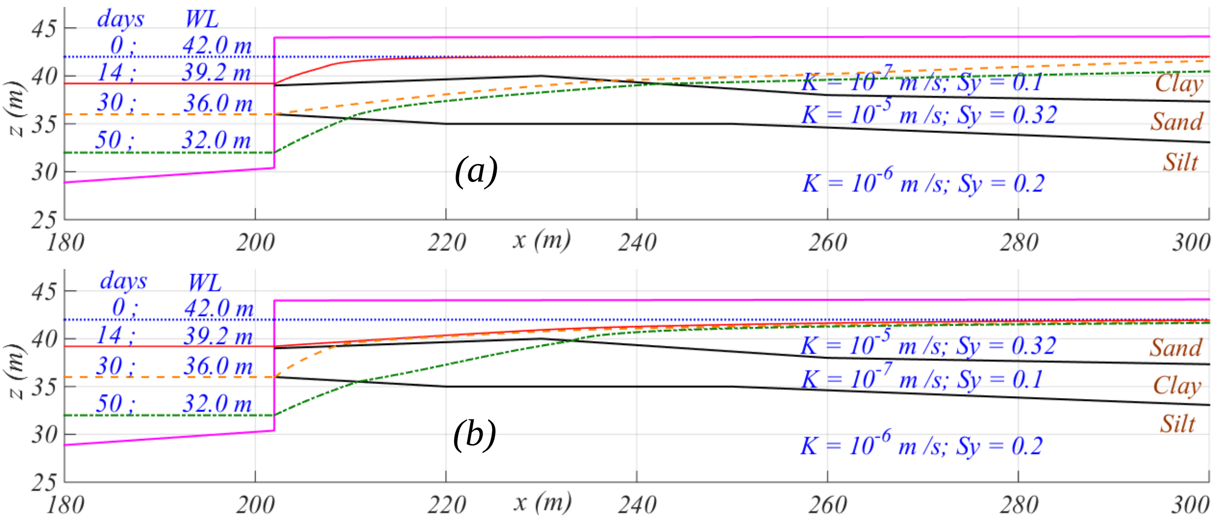

Figure 3. *Groundwater tables configurations when WL drawdown. (a) and (b) have the same geometries and WL drawdown condition, but the soil parameters of the first and second layer are reversed.*

As a conclusion, in relation to the evaluation of riverbanks stability, the proposed *1D* simplified model of transient groundwater flow through a multilayered soil shows the key role of the hydraulic conductivity of the different layers on the transient positions of the groundwater table and consequently on the corresponding pore pressure field in the soil, which influences significantly the bank stability.

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