Analysis of capillary bridges using imaging techniques and recent analytical model.

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RÉSUMÉ. Les propriétés mécaniques des matériaux granulaires non saturés sont directement liées aux interactions capillaires résultant de la présence d'eau entre les grains. Dans ce l'article, le modèle analytique récent obtenu dans [GAG 14] est utilisé pour analyser les interactions capillaires à l'échelle du pont liquide entre deux grains sphériques. Les profils géométriques des ponts capillaires sont obtenus en résolvant l'équation de Young-Laplace considérée comme un problème inverse, la pression capillaire étant inconnue. La donnée de l'angle de remplissage, de l'angle de mouillage et du rayon de gorge, qui sont déterminés expérimentalement par traitement d'image, permet de calculer toutes les données associées au pont capillaire (forme de la méridienne, volume du pont capillaire, force capillaire). Les résultats de la modélisation sont en bon accord avec les résultats expérimentaux dans le cas de petits volumes d'eau et/ou de petites distances entre les grains, où l'influence de la gravité est limitée. En revanche, pour des volumes d'eau plus importants et/ou des distances entre les grains plus grandes, l'influence de la gravité induit une distorsion du pont capillaire. Pour s'affranchir de la gravité, des essais ont été réalisés dans des conditions de micro-gravité (vol parabolique). Pour ces essais, les résultats théoriques et expérimentaux sont en bon accord, indépendamment des volumes de liquide et/ou des distances entre grains.

ABSTRACT. Mechanical properties of non-saturated granular materials are directly connected with capillary interactions, resulting from the presence of water between grains. In this paper, we use the recent analytical model obtained in [GAG 14] to analyze capillary interactions at the scale of liquid bridge between two spherical grains. Geometrical profiles of capillary bridges are obtained resolving Young-Laplace equation, considered as an inverse problem, with unknown capillary pressure. Data of contact angle, half-filling angle and gorge radius, determined experimentally by image processing, allow to calculate all the data associated with capillary bridge (form of bridge profile, Laplace pressure, capillary force). Results of theoretical modeling match very accurately experimental ones at small volumes and/or small separation distances between grains, when influence of gravity is limited. However, for larger liquid volumes and/or larger separation distances between grains the influence of the gravity is manifested as a distortion of capillary bridge. To avoid the gravity influence, experimental tests were realized in micro-gravity conditions (parabolic flight). For these tests, theoretical results are in good agreement with experimental ones, independently of liquid volumes and/or separations distances between grains.

MOTS-CLÉS : pont capillaire, Young-Laplace equation, inverse problem, mesures experimentales, microgravité

KEYWORDS: capillary bridge, Young-Laplace equation, inverse problem, experimental measurement, microgravity

1. Introduction

Capillary bridge is a liquid link between solid particles (grains). It gives origin of several phenomena observed in unsaturated granular materials, in particular of interparticle capillary force. This force contributes to the formation, deformation and flow of granular materials (soils, powders, photonic crystal production, etc.), with significant influence on the mechanical and physical properties of such materials. It imparts an apparent macroscopic strength (sand castle effect) to the moist granular materials, even in the absence of intrinsic cohesion or confining stress (see e.g. [PER 09, RAH 95, ELY 05, RIC 06]). The knowledge on the behavior of capillary bridges at a local scale (intergranular interface) seems to be indispensable to describe the phenomena observed in humid granular materials and in further modeling of such materials. The most elementary capillary bridges (between two solid grains, pendular bridge) are analyzed for over hundred years from theoretical and experimental point of view. However, the results of numerical modeling and physical experiments do not always agree, because of some simplifications and approximations introduced often into the mathematical models. This work addresses the experimental study of capillary bridge properties with use of the recent analytical calculation of bridge profile, based on solution of Young-Laplace equation as an inverse problem [GAG 14, GAG 16]. In this method, the parameters of pendular bridge and the shape of the associated meridian may be estimated using appropriate theoretical solutions. Internal capillary pressure is here recovered from experimental measurements of three geometrical parameters : gorge radius y^*, contact angle θ and half-filling angle δ (see Fig. 1a) and profiles of pendular bridges are approximated as a parts of Delaunay roulettes.

2. Experiment and theory

Series of tests on capillary bridges between two equal spherical glass grains (diameters of 8 and 10mm), with fixed water volume V (from 1 to 10 µl) and varying intergranular separation D were realized. Experimental setup consists of photo camera with support, two spherical grains with its fixation system and background light (detailed description presented in [MIE 14, MIE 15, GAG 16]). Capillary bridge with fixed volume V from 1 to 10 µl is created between two glass spheres at D=0 (grains in contact). Then, D is increased with the step of 0.1mm, up to the rupture of the bridge. At each examined D, photo of the bridge is recorded after several seconds needed for bridge stabilization. Obtained images are adjusted with image treatment softwarel *imageJ* and geometrical parameters of capillary bridge (y^*, δ, θ, Fig. 1a) are determined with use of Matlab code (see [GAG 16]). The criterion for determining the shape of capillary bridge is checked (see [GAG 14]) and bridges are parametrized using equations presented in [GAG 14]. Resulting curves (portion of nodoid or of unduloid) are superposed on the photo of capillary bridge, as presented in Fig. 2. Then associated variables (intergranular force F_{cap}, Laplace pressure Δp) are determined from analytical calculations ([GAG 14]).

3. Results

For each examined configuration, measured geometrical parameters are used to reconstruct the bridge profile and to calculate bridge characteristics. Evolution of variables associated with capillary bridge is linked directly with evolution of its geometrical parameters : y^*, δ and θ. During experiments, y^* decreases almost linearly with increasing D. Contact angle θ and half-filling angle δ initially decrease with increasing D, but θ becomes constant very soon, while δ decreases to about 50% of final separation (D at rupture), than it remains constant (see [GAG 16, MIE 15]). In result, mean bridge curvature H decreases almost continuously with increased separation (with small initial increase), it passes through zero and it becomes negative at about 70% - 90 % of the final separation D, depending on water volume V (Fig. 1b). The evolution of H determines also the evolution of Laplace pressure Δp. It is initially negative, but when H becomes negative, Δp becomes positive. Change of sign of Δp is linked directly to the change of geometry (Delaunay's roulette) describing the shape of the bridge. From the creation of the capillary bridge to some distance D ($\Delta p < 0$, $H > 0$) the meridian is a portion of nodoid, which skips to a portion of unduloid, before rupture ($\Delta p > 0$, $H < 0$).

Capillary force F_{cap} was calculated as a sum of two contributing forces : pressure resulting force $F_{\Delta p}$ and surface tension resulting force F_{ST} (see [GAG 16]). Calculated F_{ST} is always attractive, but $F_{\Delta p}$, initially attractive, becomes repulsive at positive Δp. In general, the decrease of F_{cap} is observed with increasing D, with rupture of liquid bridge at similar D for both examined sphere diameters. The capillary force at the moment of rupture is higher for larger V and it remains always attractive (Fig. 1c).

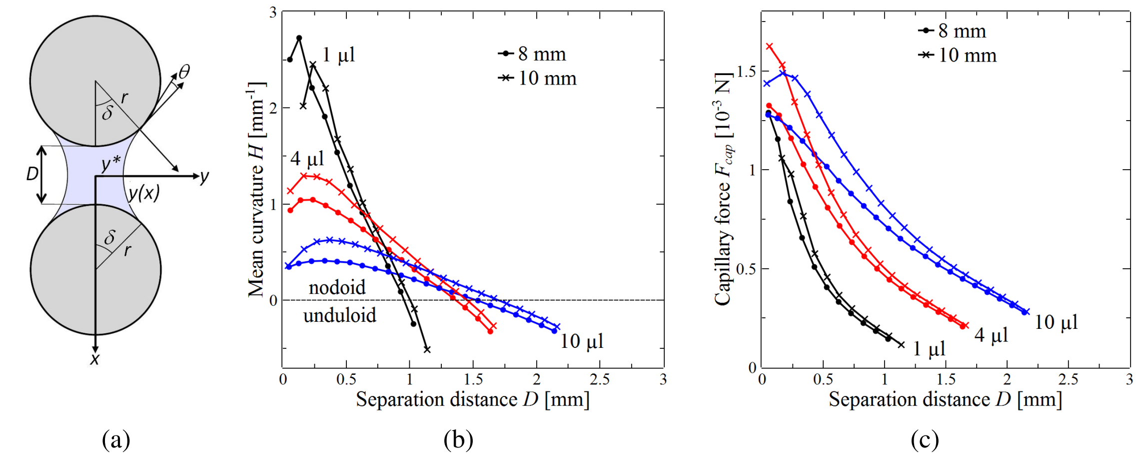
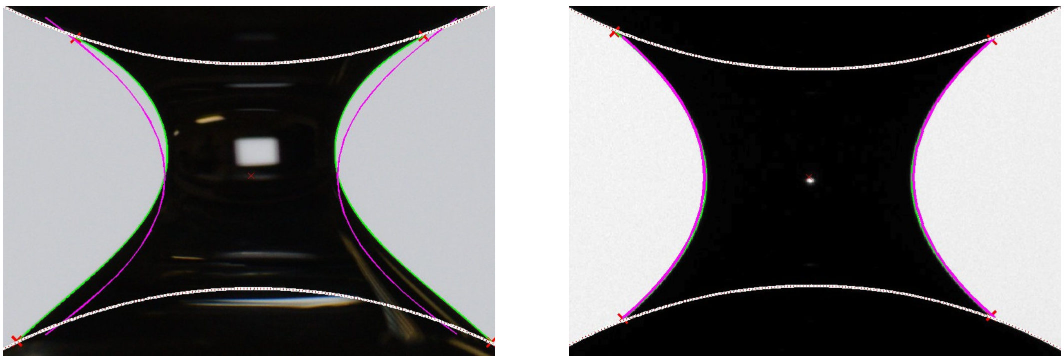


Figure 1. *Geometry of pendular capillary bridge (a), mean curvature H (b) and capillary force F_{cap} (c) as a function of distance D for capillary bridge between spheres of 8 mm and 10 mm.*



a) lab., $y^* = 0.79$ mm, $\delta = 22.2°$, $\theta = 13.5°$, unduloid   b) 0G, $y^* = 0.99$ mm, $\delta = 20.8°$, $\theta = 20.2°$, unduloid

Figure 2. *Resulting bridge profiles superposed on the original images for spheres of 10 mm. The data are for $V = 10$ µl and $D = 2.06$ mm (blue : unduloid shape, calculated from theory). The images were recorded in laboratory (a) and in microgravity conditions (b).*

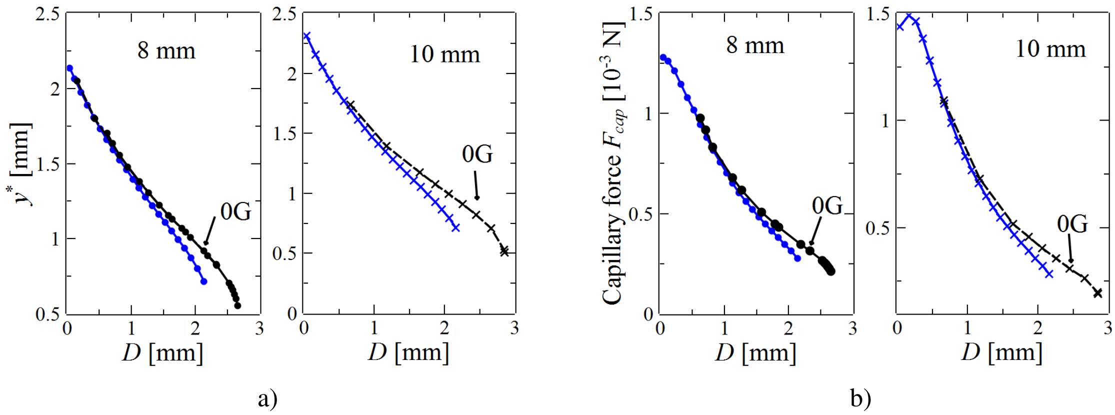


Figure 3. *Evolution of gorge radius y^* (a) and calculated capillary forces F_{cap} (b) in function of distance D for laboratory and micro-gravity conditions.*

Calculated bridge profiles are superposed on the original photos to validate the approach used in the theoretical model (see Fig. 2). For capillary bridges containing low water volume and at low separation distance between the grains (low Bond number, [GAG 16]), calculated bridge profile follows the measured one. If the volume becomes higher (higher Bond number), gravity force causes the deformation of capillary bridge, as shown in Fig. 2a. In such cases, the geometry of capillary bridges is changed and calculated variables are no more correct. To validate analytical model for higher volumes, preliminary tests were realized in micro-gravity conditions, during parabolic flight. Comparison of some results (gorge radius, capillary force) obtained in laboratory and during parabolic flight is presented in Fig. 3. Is is seen, that gorge radius y^* is higher for "zero G" tests, particularly for higher D. Also other geometrical parameters (δ, θ, H) differ significantly. In results, calculated F_{cap} (Fig. 3b) close to the rupture are larger for "zero G"-tested bridges. In addition, the rupture of capillary bridges occurs at larger D. It was noticed, that almost no changes in $F_{\Delta p}$ are visible, while F_{ST} is visibly higher for micro-gravity tests.

4. Conclusions

In this paper, we use an analytical model to determine the variables and parameters associated with pendular capillary bridge. The validity of the model was demonstrated by comparison with experimental results. Theoretical and experimental results match very accurately for small liquid volumes and small separation distances between grains. For larger ones, the capillary bridge loses its symmetry due the effect of gravity. In order to remove this effect, initial experiments in micro-gravity conditions were performed (parabolic flight, LASIE-LMGC-CNES). To avoid the gravity influence, experimental test were realized in micro-gravity conditions (parabolic flight). For these tests, theoretical results are in good agreement with experimental ones, independently of liquid volumes and/or separations distances between the grains. For examined capillary bridges, their profiles evolve from nodoid to unduloid shape with increasing D. This passage is accompanied by the increase of internal pressure, from initially negative to positive pressure at the rupture, while capillary force decreases with increasing D. Experimental validation of proposed model justifies its further development, with its adaptation to other configurations of capillary bridges (sphere-plate, plate-plate), including dynamic processes (extension, evaporation, coalescence of pendular bridges) and analysis of stability/rupture of liquid bridges, planned in the nearest future. Enhanced analytical model will be also validated experimentally in laboratory and in micro-gravity conditions. It would constitute a base for further model upscaled to macroscopic scale (sample scale).

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