Enhancement of multifiber beam elements in the case of reinforced concrete structures for taking into account the lateral confinement of concrete due to stirrup

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RÉSUMÉ. *Pour déterminer la vulnérabilité sismique des structures existantes en béton armé, des méthodes de calcul numérique à l'échelle structurelle efficaces et suffisamment précises sont nécessaires. Dans le cas d'éléments poutres, les recherches actuelles concernent le développement de cinématiques enrichies pour mieux tenir compte de la déformée des sections. A cette échelle, des formulations permettent déjà de reproduire les déformations de cisaillement. En revanche, concernant les cadres d'armatures, soit ils ne sont pas du tout considérés, soit ils ne permettent pas d'établir un véritable équilibre global de la section transversale permettant de calculer le confinement du béton. Cependant, comme le montrent certains essais expérimentaux, la quantité d'armatures transversales pilote d'une manière significative le comportement de ces poutres. Le travail présenté dans cet article propose un modèle poutre multifibre enrichi pour la prise en compte de l'effet de confinement du béton par les armatures transversales. Des déplacements transversaux additionnels d'enrichissement permettent d'enrichir la cinématique de la section transversale pour y calculer l'équilibre. Le modèle est dans un premier temps validé avec des tests simples et une loi de comportement élastique linéaire pour le matériau composant la section.*

ABSTRACT. *To assess the seismic vulnerability of existing reinforced concrete structures, a large number of degrees of freedom is involved. Consequently, efficient numerical tools are required. In the case of slender elements, enhanced beam elements have been developed to try to introduce shear effects, but in these models, the transverse steel is sometimes taken into consideration with approximated manner or often not at all. However, as shown by some experimental tests, the amount of transverse reinforcement triggers significantly the behavior of beam elements, especially under cyclic loading. Thus, the main goal of this work is to investigate solutions for an enhanced multifiber beam element accounting for vertical stretching of the cross section, occurring due to the presence of transverse reinforcement. The efficiency of the proposed modeling strategies is tested with results obtained from tension and flexure tests conducted on an elastic linear material.*

MOTS-CLÉS : *confinement, aciers transversaux, éléments finis, élément poutre multifibre, béton armé.*

KEYWORDS: *confinement, stirrups, finite elements, multifiber beam elements, reinforced concrete.*

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1. Introduction

To study the seismic vulnerability of existing reinforced concrete structures, numerical computations at the structural scale able to account for material non-linearities are needed. An alternative to full solid models, which are too costly, is the use of multifiber beam elements. The latter one combines the advantages of high computational speed with an increased accuracy for nonlinear materials. It consists on adding a two dimensional section at the Gauss point of the element.
In addition, a variety of approaches have been developed to try to introduce shear effects, such as those proposed by [LEC 12], but whose model can't be applied to reinforced concrete elements, and the numerical formulation of [MOH 10] which is adapted to reinforced concrete applications but works only in 2D. More recently, [CAP 16] developed a nonlinear multifiber beam model which provides robust results by the introduction of warping in the case of reinforced concrete beams. In the above mentioned works, the transverse steel is sometimes taken into consideration with approximated manner or often not at all. However, as shown by some experimental tests conducted by [CUS 95], the amount of transverse reinforcement triggers significantly the behavior of beam elements, especially under cyclic loading. Thus, the main goal of this work is to investigate solutions for an enhanced multifiber beam element accounting for vertical stretching of the cross section, occurring due to the presence of transverse reinforcement.

2. Formulation of the enhanced multifiber beam element

A 2D multifiber Timoshenko beam, displacement-based, element has been developed. The main hypothesis considered herein is that the full displacement of any fiber in the cross-section can be approximated by the sum of the plane section displacement field ($\mathbf{u}^P$), obtained from Timoshenko's beam theory, and a new displacement field that enables the section to distort and warp ($\mathbf{u}^w$). The distortion field has one transverse component, u_y^w, which stands for the distortion of the section. Thus, the total displacement at any material point of the cross section is given by the following equation :

$$\mathbf{u} = \mathbf{u}^P + \mathbf{u}^w = \underbrace{\begin{bmatrix} U_x - y\theta_z \\ U_y \end{bmatrix}}_{u^P} + \underbrace{\begin{bmatrix} 0 \\ u_y^w(x,y) \end{bmatrix}}_{u^w} \tag{1}$$

The complete strain will be formed by the sum of the plane strain field ϵ^P and the warp strain ϵ^w. Following on from the kinematics given by equation 1, the linearised strain tensor reads :

$$\epsilon = \frac{1}{2}\left(grad(u) + grad(u)\right)^T = \epsilon^{\mathbf{P}}(\mathbf{u}^P) + \epsilon^{\mathbf{w}}(\mathbf{u}^w) \tag{2}$$

Equation 3 expresses the non linear constitutive law :

$$\sigma = \hat{\sigma}(\epsilon^p, \epsilon^w) \tag{3}$$

2.1. *Governing equations*

Beam equilibrium is written in its weak form by equation 4. In addition, the plane section displacement u^p and the distortion one u^w assumed to be orthogonal, the projection of the PPV* on these two subspaces lead to two equilibrium equations (equation 5). The first one representing the classical equilibrium of the beam element, and the second one being the equilibrium equation of the cross section. F denotes the external forces and P^w the forces coming from constrained warping at the beam ends.

$$\int_\Omega \epsilon^{*T} \sigma d\Omega = U^{*T} F_{ext} \tag{4}$$

$$\Leftrightarrow \begin{cases} \displaystyle\int_\Omega \delta\epsilon^{pT} \hat{\sigma}(\epsilon^p, \epsilon^w) d\Omega &= F \\ \displaystyle\int_\Omega \delta\epsilon^{wT} \hat{\sigma}(\epsilon^p, \epsilon^w) d\Omega &= P^w \end{cases} \tag{5}$$

2.2. *Spatial discretization*

The deformations of the plane section ϵ^P, can be expressed as a function of the generalized deformations, e_s, as shown in equation 6 :

$$\epsilon^P = \begin{bmatrix} \epsilon^P_{xx} \\ \epsilon^P_{yy} \\ \gamma^P_{xy} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & -y \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{a_s(y)} \underbrace{\begin{bmatrix} \frac{dU_x}{dx} \\ \frac{dU_y}{dx} - \theta_z \\ \frac{d\theta_z}{dx} \end{bmatrix}}_{e_s} \tag{6}$$

$$\epsilon^P = a_s(y) B_p U^e \tag{7}$$

The displacements of a standard 2D beam element consist of two translations u and v, in x and y directions respectively, and one rotation θ_z about z axis. These are functions of position x along the element axis and are collected in displacement vector U^e. B_p gathers the derivatives of the interpolation functions related to longitudinal spatial discretization. The use of linear interpolation functions leads to shear locking problems. Recently, a new multifiber beam element, with internal degrees of freedom, and higher-order interpolation functions has been developed by [CAI 15] and has been chosen to be introduced in our model.

In addition, the 2D beam element, in the present study, is enhanced with a warping-distortion displacement u^w_y, given by the following expression :

$$u^w(x, y) = u^w_y = c(x)\varphi(y)W^e \tag{8}$$

Hence, the warping strains can be written in the form :

$$\begin{bmatrix} \epsilon^w_{xx} \\ \epsilon^w_{yy} \\ \gamma^w_{xy} \end{bmatrix} = \begin{bmatrix} \frac{\partial u^w_x}{\partial x} \\ \frac{\partial u^w_y}{\partial y} \\ \frac{\partial u^w_x}{\partial y} + \frac{\partial u^w_y}{\partial x} \end{bmatrix} = \begin{bmatrix} 0 \\ c(x)\frac{\partial \varphi(y)}{\partial y} \\ \frac{dc(x)}{dx}\varphi(y) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & \frac{\partial \varphi}{\partial y} \\ \varphi & 0 \end{bmatrix}}_{a_w} \underbrace{\begin{bmatrix} \frac{dc}{dx} \\ c \end{bmatrix}}_{B_w W^e} \tag{9}$$

Where W_e gathers the warping degrees of freedom, and B_w the longitudinal interpolation functions and their derivatives. As for the components of a_s and a_w, they are computed at the Gauss points of each section which is discretized by using six-nodes triangular elements. After discretization of the beam and the section domains, the formulation leads to two coupled systems of nonlinear equations to solve for the classical beam nodal displacements and for the additional warping displacements.

3. Results

A simple cantilever beam submitted to transversal diplacement and then to an axial displacement has been modelled numerically by using 30 Timoshenko multifiber beam elements. The cross section of the beam is considered to be 0.15 m $\times$ 0.15 m area and is meshed using 72 six-nodes quadratic triangles (TRI6). It is made of an homogeneous material considered to be elastic linear.

Figure 1 shows the variation of transversal stresses σ_{yy} computed at each fiber, with respect to the length of the beam, as well as section maps of σ_{yy} obtained at different positions : next to the fixed and free ends. A 2D model meshed with 492 TRI6 elements has also been conducted in order to compare the variation of the lateral normal stresses with the enhanced multifiber beam element. Also, the evolution of transverse displacement's maps are displayed to the right of figure 1 and the maximum local error for transverse displacement is of the order of 0.011% for flexure. These results prove that the equilibrium state is reached and the enhanced developed model works well in the linear elastic phase.

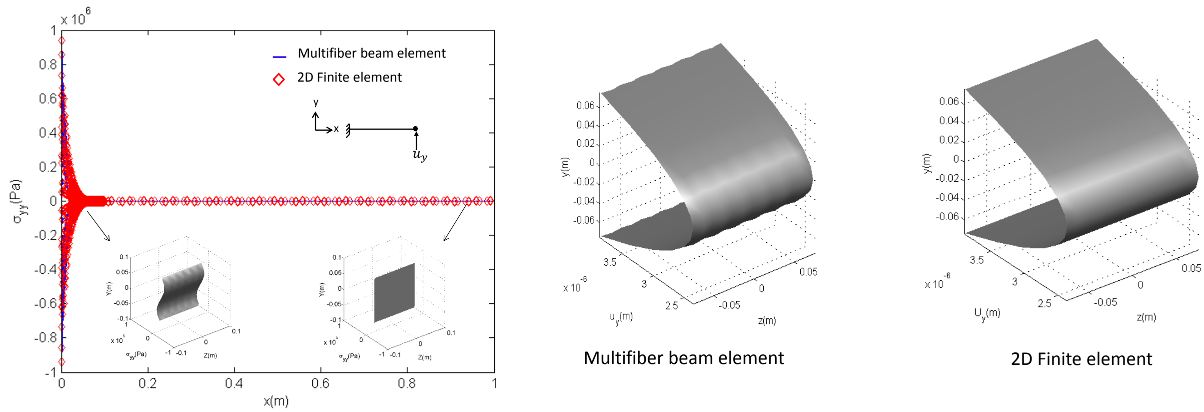

Figure 1. *To the left : Variation of the transversal stresses σ_{yy} with respect to the length of the beam subjected to flexure test. Section maps of σ_{yy} at different positions of the beam. To the right : transverse displacement maps obtained with the enhanced model and the 2D finite element*

4. Conclusion

This article proposes a method to account for vertical stretching of the cross section. The efficiency of the proposed modeling strategies is tested with results obtained from tension and flexure tests conducted on an homogeneous elastic linear material. The results prove that the equilibrium state is reached and the enhanced developed model works well in the linear elastic phase. The ongoing works aim at modelling transverse steel as well as implementing an existing constitutive law for concrete under monotonic and cyclic loadings and hence study the non-linear response of structural elements subjected to transverse shear.

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