Analysis and modeling of interfacial stresses in RC beams reinforced by external bonding with a functionally graded material plate

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RÉSUMÉ. *Dans cet article, une analyse des contraintes d'interface est présentée pour une poutre en béton simplement supportée collé extérieurement par une plaque en matériau à gradient de propriétés. Cette nouvelle solution est destinée à être appliquée à des poutres en tous types de matériaux collés avec une plaque mince, tandis que toutes les solutions existantes ont été développées en mettant l'accent sur le renforcement des poutres en béton armé, ce qui a permis l'omission de certains termes. Il est montré que les contraintes normales et de cisaillement à l'interface sont influencées par les paramètres de matériau de renforcement et de la géométrie de la poutre. Cette recherche est utile pour comprendre le comportement mécanique de l'interface et la conception des structures hybrides FGM – RC.*

ABSTRACT. *In this paper, an interfacial stress analysis is presented for simply supported concrete beam bonded with a functionally graded material plate. This new solution is intended for application to beams made of all kinds of materials bonded with a thin plate, while all existing solutions have been developed focusing on the strengthening of reinforced concrete beams, which allowed the omission of certain terms. It is shown that both the normal and shear stresses at the interface are influenced by the material and geometry parameters of the composite beam. This research is helpful for the understanding on mechanical behavior of the interface and design of the FGM–RC hybrid structures.*

MOTS-CLÉS : *poutre en béton armé, contraintes d'interface, renforcement., plaque en matériaux à gradient de propriétés .*

KEY WORDS: *RC beam; interfacial stresses; Strengthening; Functionally graded material plate .*

1. Introduction

The objective of this theoretical study is to improve the famous method developed by Tounsi [TOU 2006] by incorporating with the adherend shear deformations. Indeed, it is reasonable to assume that the shear stresses, which develop in the adhesive, are continuous across the adhesive–adherend interface. The importance of including shear-lag effect of the adherents was shown by Rabahi [RAB 2016] and Bouakaz [BOU 2014]. The obtained results are in good agreement with those of numerical results. The presented method predicts also, maximum values in cut-off section, but comparatively, the computed interfacial stresses are considerably smaller than those obtained by other models which neglect adherent shear deformations. Hence, the adopted improved model describes better actual response of the FGM–RC hybrid beams and permits the evaluation of the interfacial stresses, the knowledge of which is very important in the design of such structures. In this paper, the details of the interfacial shear and normal stress are analyzed by the theoretical method. The effects of the material and geometry parameters on the interface stresses are considered and compared with that resulting from literature. Finally, some concluding remarks are summarized in conclusion. It is believed that the present results will be of interest to civil and structural engineers and researchers.

2. Analytical approach

2.1- Assumptions

The present analysis takes into consideration the transverse shear stress and strain in the beam and the plate but ignores the transverse normal stress in them. One of the analytical approach proposed by Hassaine Daouadji , [HAS 2016] for concrete beam strengthened with a bonded FGM Plate [HAS 2013] (Fig 1) was used in order to compare it with a finite element analysis. The analytical approach [HAS 2016] is based on the following assumptions:

- Elastic stress strain relationship for concrete, FGM and adhesive;
- There is a perfect bond between the composite plate and the beam;
- The adhesive is assumed to only play a role in transferring the stresses from the concrete to the composite plate reinforcement;
- The stresses in the adhesive layer do not change through the direction of the thickness.

Since the functionally graded materials is an orthotropic material. In analytical study [HAS 2016], the classical plate theory is used to determine the stress and strain behaviours of the externally bonded composite plate in order to investigate the whole mechanical performance of the composite – strengthened structure.

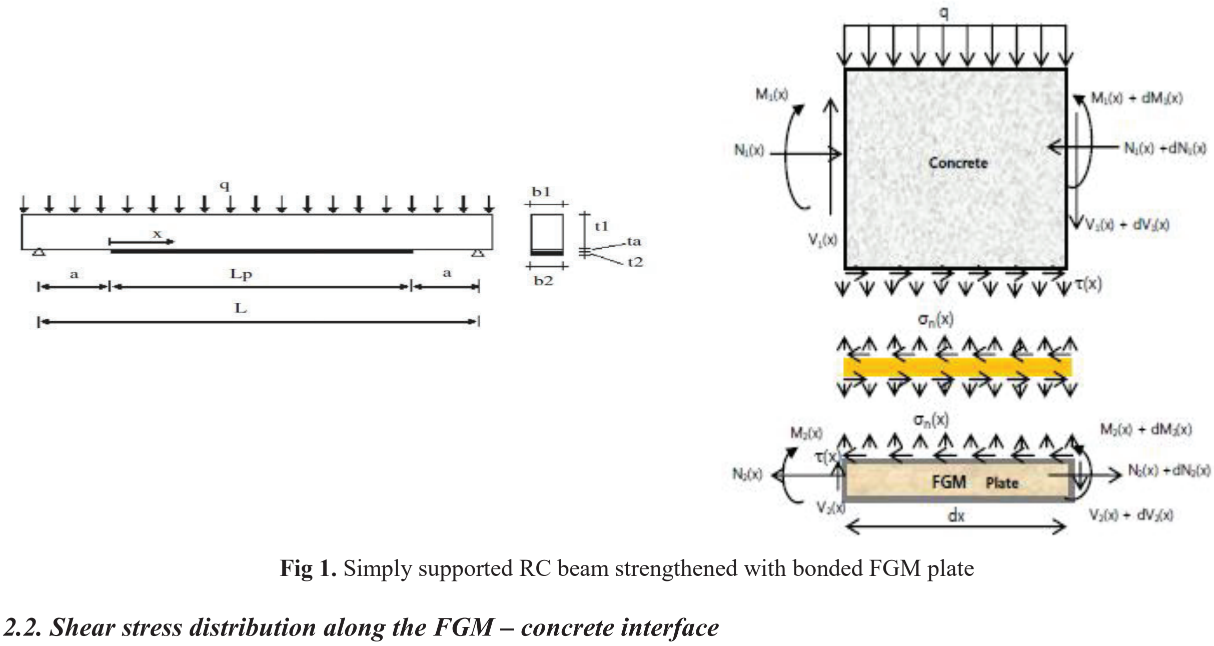

Fig 1. Simply supported RC beam strengthened with bonded FGM plate

2.2. Shear stress distribution along the FGM – concrete interface

The governing differential equation for the interfacial shear stress [HAS 2016] is expressed as:

$$\frac{d^2\tau(x)}{dx^2} - K_1\left(A'_{11} + \frac{b_2}{E_1A_1} + \frac{(y_1+t_2/2)(y_1+t_a+t_2/2)}{E_1I_1D'_{11}+b_2}b_2D'_{11}\right)\tau(x) + K_1\left(\frac{(y_1+t_2/2)}{E_1I_1D'_{11}+b_2}D'_{11}\right)V_T(x)=0 \qquad (1)$$

where:

$$K_1 = \frac{1}{\left(\frac{t_a}{G_a} + \frac{t_1}{4G_1}\right)} \qquad (2)$$

For simplicity, the general solutions presented below are limited to loading which is either concentrated or uniformly distributed over part or the whole span of the beam, or both. For such loading, $d^2V_T(x)/dx^2 = 0$, and the general solution to Eq. (1) is given by

$$\tau(x) = B_1 \cosh(\lambda x) + B_2 \sinh(\lambda x) + m_1 V_T(x) \tag{3}$$

where:

$$\lambda^2 = K_1\left(A_{11}^{'} + \frac{b_2}{E_1 A_1} + \frac{(y_1 + t_2/2)(y_1 + t_a + t_2/2)}{E_1 I_1 D_{11}^{'} + b_2} b_2 D_{11}^{'} \right) \tag{4}$$

$$m_1 = \frac{K_1}{\lambda^2}\left(\frac{(y_1 + t_2/2)}{E_1 I_1 D_{11}^{'} + b_2} D_{11}^{'} \right) \tag{5}$$

And B_1 and B_2 are constant coefficients determined from the boundary conditions. In the present study, a simply supported beam has been investigated which is subjected to a uniformly distributed load (Fig 1). The interfacial shear stress for this uniformly distributed load at any point is written as [HAS 2016]:

$$\tau(x) = \left[\frac{m_2 a}{2}(L-a) - m_1 \right] \frac{q e^{-\lambda x}}{\lambda} + m_1 q\left(\frac{L}{2} - a - x \right) \qquad 0 \le x \le L_P \tag{6}$$

Where q is the uniformly distributed load and x; a; L and L_p are defined in Fig. 1.

2.3. Normal stress distribution along the FGM – concrete interface

The following governing differential equation for the interfacial normal stress [HAS 2016]:

$$\frac{d^4\sigma_n(x)}{dx^4} + K_n\left(D_{11}^{'} + \frac{b_2}{E_1 I_1} \right)\sigma_n(x) - K_n\left(D_{11}^{'}\frac{t_2}{2} - \frac{y_1 b_2}{E_1 I_1} \right)\frac{d\tau(x)}{dx} + \frac{qK_n}{E_1 I_1} = 0 \tag{7}$$

The general solution to this fourth–order differential equation is

$$\sigma_n(x) = e^{-\beta x}\left[C_1 \cos(\beta x) + C_2 \sin(\beta x) \right] + e^{\beta x}\left[C_3 \cos(\beta x) + C_4 \sin(\beta x) \right] - n_1 \frac{d\tau(x)}{dx} - n_2 q \tag{8}$$

For large values of x it is assumed that the normal stress approaches zero and, as a result, $C_3 = C_4 = 0$. The general solution therefore becomes

$$\sigma_n(x) = e^{-\beta x}\left[C_1 \cos(\beta x) + C_2 \sin(\beta x) \right] - n_1 \frac{d\tau(x)}{dx} - n_2 q \tag{9}$$

where :

$$\beta = \sqrt[4]{\frac{K_n}{4}\left(D_{11}^{'} + \frac{b_2}{E_1 I_1} \right)} \qquad n_1 = \left(\frac{y_1 b_2 - D_{11}^{'} E_1 I_1 t_2 / 2}{D_{11}^{'} E_1 I_1 + b_2} \right) \qquad n_2 = \frac{1}{D_{11}^{'} E_1 I_1 + b_2} \tag{10}$$

As is described by Hassaine Daouadji [HAS 2016], the constants C_1 and C_2 in Eq.(9) are determined using the appropriate boundary conditions and they are written as follows:

$$C_1 = \frac{K_n}{2\beta^3 E_1 I_1}\left[V_T(0) + \beta M_T(0) \right] - \frac{n_3}{2\beta^3}\tau(0) + \frac{n_1}{2\beta^3}\left(\frac{d^4\tau(0)}{dx^4} + \beta\frac{d^3\tau(0)}{dx^3} \right) \tag{11}$$

$$C_2 = -\frac{K_n}{2\beta^2 E_1 I_1} M_T(0) - \frac{n_1}{2\beta^2}\frac{d^3\tau(0)}{dx^3} \tag{12}$$

$$n_3 = b_2 K_n\left(\frac{y_1}{E_1 I_1} - \frac{D_{11}^{'} t_2}{2 b_2} \right) \tag{13}$$

The above expressions for the constants C_1 and C_2 has been left in terms of the bending moment $M_T(0)$ and shear force $V_T(0)$ at the end of the soffit plate. With the constants C_1 and C_2 determined, the interfacial normal stress can then be found using Eq.(9).

3. Numerical verification and discussions:

3.1. Comparison with approximate solutions:

The present simple solution is compared, in this section, with some approximate solutions available in the literature. These include Smith and Teng [SMI 2001], Tounsi [TOU 2008] and Hassaine Daouadji [HAS 2016] solutions uniformly distributed loads. A comparison of the interfacial shear and normal stresses from the different existing closed – form solutions and the present solution is undertaken in this section. An undamaged beams bonded with CFRP, FGM and sandwich (P-FGM core) plate soffit plate is considered. The beam is simply supported and subjected to a uniformly distributed load. A summary of the geometric and material properties is given in table 1. The results of the peak interfacial shear and normal stresses are given in table 2 for the beams strengthened by bonding CFRP, FGM and sandwich (P-FGM core) plate. As it can be seen from the results, the peak interfacial stresses assessed by the present theory are smaller compared to those given by Smith and Teng [SMI 2001], Tounsi [TOU 2008] and Hassaine Daouadji [HAS 2016] solutions. This implies that adherend shear deformation is an important factor influencing the adhesive interfacial stresses distribution. Fig 2 plots the interfacial shear and normal stresses near the plate end for the example RC beam bonded with a FGM plate for the uniformly distributed load case. Overall, the predictions of the different solutions agree closely with each other. The interfacial normal stress is seen to change sign at a short distance away from the plate end. The present analysis gives lower maximum interfacial shear and normal stresses than those predicted by Tounsi [TOU 2008] and Hassaine Daouadji [HAS 2016], indicating that the inclusion of adherend shear de formation effect in the beam and soffit plate leads to lower values of σ_{max} and σ_{max}. However, the maximum interfacial shear and normal stresses given by Tounsi [TOU 2008] and Hassaine Daouadji [HAS 2016] methods are lower than the results computed by the present solution. This difference is due to the assumption used in the present theory which is in agreement with the beam theory. Hence, it is apparent that the adherend shear deformation reduces the interfacial stresses concentration and thus renders the adhesive shear distribution more uniform. The interfacial normal stress is seen to change sign at a short distance away from the plate end.

Table 1: Geometric and material properties.

Component	Width (mm)	Depth (mm)	Young's modulus (MPa)	Poisson's ratio	Shear modulus (MPa)
RC beam	b_1 = 200	t_1=300	E_1 = 30 000	0.18	-
Adhesive layer RC beam	b_a = 200	t_a = 4	E_a = 3000	0.35	-
GFRP plate (bonded RC beam)	b_2 = 200	t_2= 4	E_2 = 50 000	0.28	G_{12} =5000
CFRP plate (bonded RC beam)	b_2 = 200	t_2= 4	E_2 = 140 000	0.28	G_{12} =5000
FGM plate (bonded RC beam)	b_2 = 200	t_2= 4	E_2 = 380 000	0.3	G_{12} =5000

Table 2: Comparison of peak interfacial shear and normal stresses (MPa): Uniformly Distributed Load- UDL

Model	Shear	Normal
Tounsi [TOU 2008] "RC beam with CFRP plate"	1.96203	1.1694
Smith and Teng [SMI 2001] "RC beam with CFRP plate"	3.8347	2.1012
Hassaine Daouadji [HAS 2016] "RC beam with CFRP plate"	1.9982	1.1887
Present Model "RC beam with P-FGM plate" P-FGM - Al_2O_3 (P=2)	2.1573	1.1939

The results of the peak interfacial shear and normal stresses are given in table 2 for the RC beam with a CFRP, FGM and sandwich soffit plate. Table 2 shows that, for the UDL case, the present solution gives results which generally agree better with those from Smith's and Teng [SMI 2001], Tounsi's [TOU 2008] and Hassaine Daouadji's [HAS 2016] solutions. The latter two again give similar results. In short, it may be concluded that all solutions are satisfactory for RC beams bonded with a thin plate as the rigidity of the soffit plate is small in comparison with the that of the RC beam. Those solutions which consider the additional bending and shear deformations in the soffit plate due to the interfacial shear stresses give more accurate results. The present solution is the only solution which covers the uniformly distributed loads and considers this effect and the effects of other parameters.

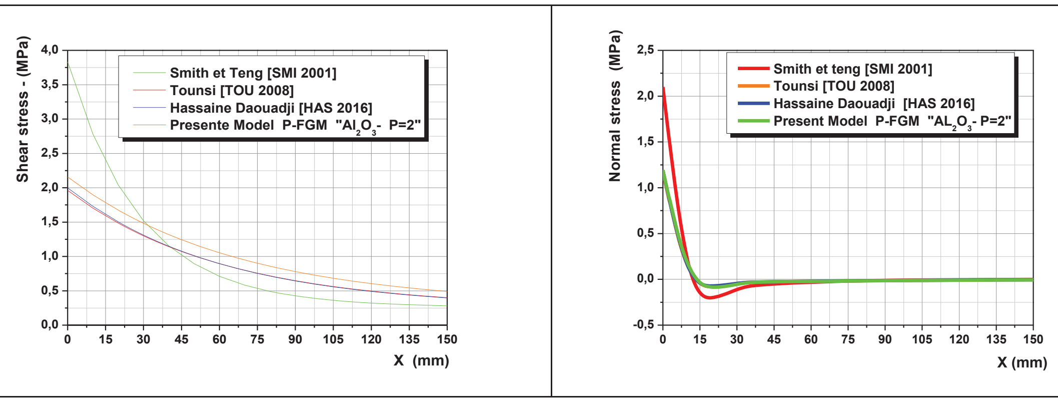

Fig 2. Comparison of interfacial shear and normal stress for P-FMP plated RC beam with the analytical results:
(a) Shear stress (b) Normal stress

3.2. Effect of plate stiffness on interfacial stress:

Table 3: Comparison of peak interfacial shear and normal stresses: Effect of plate stiffness

Material	Shear stress	Normal stress
CFRP	1.9620	1.1694
GFRP	1.1943	0.8995
Steel	2.3326	1.2795
P-FGM - (P=2)	2.1573	1.1939

Table 4: Effect of plate stiffness on interfacial stresses in FGM strengthened RC beam

Material	P-FGM						
	Ceramic: P = 0	P = 0,5	P = 2	P = 5	P = 10	P = 100	Aluminum: P = ∞
Shear stress	2.9391	2.6158	2.1573	1.8398	1.6635	1.4420	1.4120
Normal stress	1.3972	1.3484	1.1939	1.0658	1.0064	0.9790	0.9855

Table 3 and 4 gives interfacial normal and shear stresses for the RC beam bonded with a CFRP, GFRP, steel and FGM plate, respectively, which demonstrates the effect of plate material properties on interfacial stresses. The length of the plate is Lp=2400mm, and the thickness of the plate and the adhesive layer are both 4 mm. The results show that, as the plate material becomes softer (from FGM to steel, to CFRP and then GFRP), the interfacial stresses become smaller, as expected. This is because, under the same load, the tensile force developed in the plate is smaller, which leads to reduced interfacial stresses. The position of the peak interfacial shear stress moves closer to the free edge as the plate becomes less stiff. Effect of plate stiffness on interfacial stress : Fig. 3 and 4 gives interfacial normal and shear stresses for the RC beam bonded with a FGM plate, respectively, which demonstrates the effect of plate material properties on interfacial stresses. The length of the plate is Lp=2400 mm, and the thickness of the plate and the adhesive layer are both 4mm. The results show that, as the plate material becomes softer, the interfacial stresses become smaller, as expected. This is because, under the same load, the tensile force developed in the plate is smaller, which leads to reduced interfacial stresses. The position of the peak interfacial shear stress moves closer to the free edge as the plate becomes less stiff.

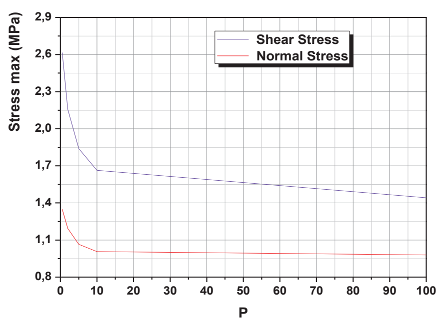

Fig.3. Effect of degree of homogeneity on interfacial stresses in P-FGM (P = 2) strengthened RC beam

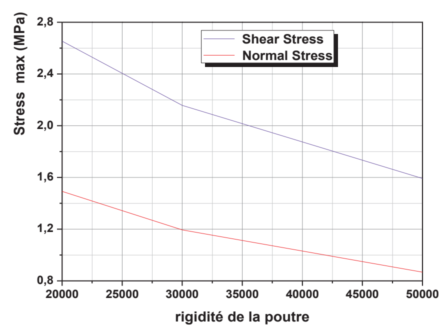

Fig.4. Effect of plate stiffness on interfacial stresses in FGM strengthened RC beam.

3.3. Effect of length of unstrengthened region a:

The influence of the length of the ordinary-beam region (the region between the support and the end of the composite strip on the edge stresses) appears in fig 5. It is seen that, as the plate terminates further away from the supports, the interfacial stresses increase significantly. This result reveals that in any case of strengthening, including cases where retrofitting is required in a limited zone of maximum bending moments at midspan, it is recommended to extend the strengthening strip as possible to the lines.

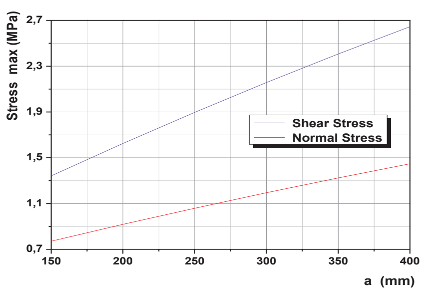

Fig.5. Influence of length of unstrengthened region on edge stresses for RC beam with a bonded P-FGM soffit plate

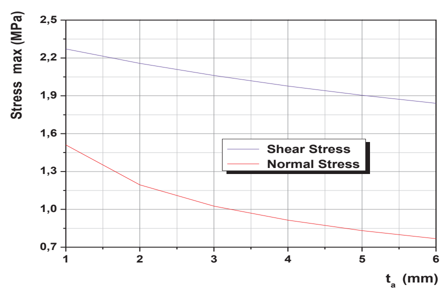

Fig.6. Effect of adhesive layer thickness on edge shear stresses in P-FGM strengthened RC beam.

3.4. Effect of adhesive layer thickness: Fig. 6 show the effects of the thickness of the adhesive layer on the interfacial stresses. Increasing the thickness of the adhesive layer leads to a significant reduction in the peak interfacial stresses. Thus using thick adhesive layer, especially in the vicinity of the edge, is recommended. In addition, it can be shown that these stresses decrease during time, until they become almost constant after a very long time.

4. Conclusion

This paper has been concerned with the prediction of interfacial shear and normal stresses in reinforced concrete beams strengthened by externally bonded functionally graded FGM plate. Such interfacial stresses provide the basis for understanding debonding failures in such RC beams and for the development of suitable design rules. It is shown that the in homogeneities play an important role in interfacial stresses. The obtained solution could serve as a basis for establishing simplified FGM theories or as a benchmark result to assess other approximate methodologies. we can conclude that, This research is helpful for the understanding on mechanical behavior of the interface and design of the FGM–RC hybrid structures. The new solution is general in nature and may be applicable to all kinds of materials.

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