Computation of energy free ratio for interfacial crack stuck between different isotropic materials by mixed finite element

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RÉSUMÉ. *Dans la présente communication, nous présentons la formulation d'un élément fini mixte basé sur le principe de variation de Reissner dont le but de son utilisation pour modéliser les interfaces fissurées entre deux matériaux différents. Cet élément d'interface est développé sur la base de la méthode d'extension de fissure virtuelle pour évaluer le taux d'énergie libre. Les résultats obtenus à partir du présent élément fini mixte d'interface se sont révélés être en bon accord avec les solutions analytiques pour les biomatériaux isotropes. Le comportement à la rupture des fissures d'interface entre matériaux dissemblables est un problème très important des matériaux composites. Williams en 1959 a effectué une analyse asymptotique du champ élastique à la pointe de fissure ouverte et il a découvert que le champ de contraintes possède un caractère oscillatoire. Il a d'abord étudié le problème d'une fissure inter faciale entre deux matériaux isotropes dissemblables. Erdogan en 1963, England en 1965 et Rice et Sih en 1965 présentent les solutions asymptotiques du champ de contraintes autour de la pointe d'une fissure à l'interface du bi-matériaux. L'élément fini mixte développé par Bouziane et al en 2009 est utilisé pour modéliser les interfaces fissurées entre deux matériaux différents. Cet élément d'interface a été associé à la méthode d'extension de fissure virtuelle pour évaluer le taux de restitution d'énergie en utilisant un seul maillage par éléments finis. La précision de l'élément est évaluée en comparant la solution numérique à la solution analytique existante ou à des solutions numériques obtenues à partir d'éléments finis. Les résultats obtenus à partir de l'élément d'interface mixte actuel se sont confirmés être en harmonie avec les solutions analytiques.*

ABSTRACT. *In this communication a mixed finite element, based on Reissner's mixed variationally principle, is used to model the cracked interfaces between two different materials. This interface element was associated with the virtual crack extension method to evaluate the energy free ratio. Results obtained from the presently mixed interface element shown to be in good agreement with the analytical solutions for isotropic biomaterials. Fracture behavior of interface cracks between dissimilar materials is a very important problem of composite materials. Williams who performed an asymptotic analysis of the elastic field at the tip of an open crack and found that the stress field possesses an oscillatory character first investigated the problem of an interfacial crack between two dissimilar isotropic materials. Erdogan (1963), England (1965), Rice and Sih (1965) present the asymptotic solutions of the stress field around the tip of a bi-material interface crack. The mixed finite element developed by Bouziane and al (2009) is used to model the cracked interfaces between two different materials. This interface element was associated with the virtual crack extension method to evaluate the energy release rate using only one meshing by finite elements. The accuracy of the element is evaluated by comparing the numerical solution with the existing analytical solution or numerical ones gotten from others finite elements. Results obtained from the presently mixed interface element shown to be in the good settlement with the analytical solutions.*

MOTS-CLÉS : *Taux de restitution d'énergie, Méthode d'extension virtuelle de la fissure, Eléments finis mélangés, Interface fissurée, bi-matériaux.*

KEY WORDS: *Energy release rate, Virtual crack extension method, mixed finite element, cracked interface, Bi-materials.*

1. Mixed finite element and Virtual crack extension method

The present element is a 7-node two dimensional mixed finite element: 5 displacements nodes with two degrees of freedom (u_1,u_2) per node and 2 stress nodes with two degrees of freedom (σ_{22},σ_{12}) per node. The formulation start from the parent element in a natural plane with an aim of modeling different types of interfaces with various orientations. The final configuration of the element is obtained by the three following stages.

1. Construction of a parent mixed finite element;

2. Delocalization of certain variables inside the element and by displacement of static nodal unknown of the corners towards the side itself;

3. Static condensation of the internal unknown variables.

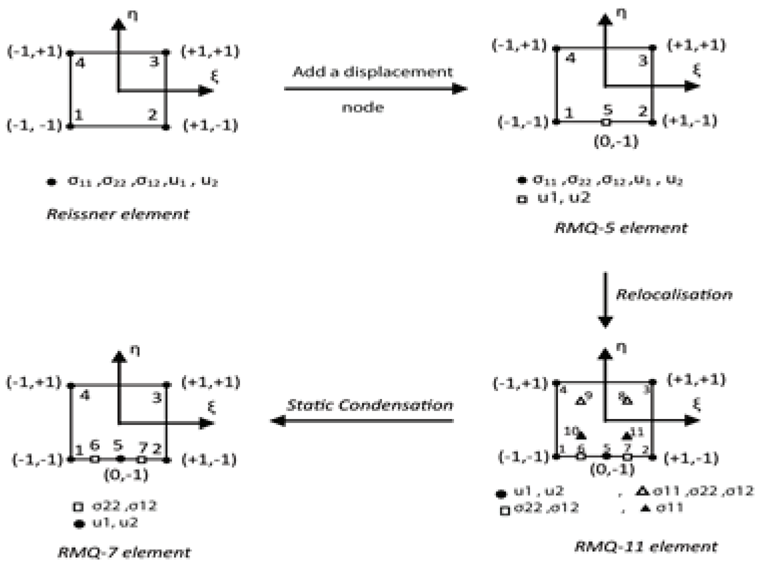

Figure 1. Stages of construction of the mixed finite element

The virtual crack extension method calculate the energy release rate:

$$G = -\frac{dU}{da} = -\frac{1}{2}\sum_{i=1}^{ne}\{u\}_i^t \frac{\Delta K_i}{\Delta a}\{u\}_i$$

where U is the potential energy of the system, (a) is the length of the crack, (Δa) is the length of the virtual crack extension, and (ΔK_i) and $\{u\}_i$ are the difference of the stiffness matrixes and the nodals displacements vectors of the element i, surrounding a crack tip at the virtual crack extension, respectively. The evaluation of G by the virtual crack extension method requires two finite element analyses.

The use of the RMQ-7 element makes it possible to introduce only one mesh for the calculation of the energy release rate. The intermediate displacement node (node 5) of the RMQ-7 element is associated to the crack tip, and consequently the length of crack (a) can be increased by a quantity (Δa) while acting inside strict of the crack element by translation of the node of crack tip without disturbing the remainder of the mesh [BOU 92].

In the used mesh, another equivalent element to that placed on the crack is employed. This element has the same geometry and it is consisted of the same material as show in figure 2. The energy release rate is calculated starting from the difference of the elementary matrices of the element containing the crack representing the state ($a+\Delta a$) and its equivalent element is representing the state a [BOU 92].

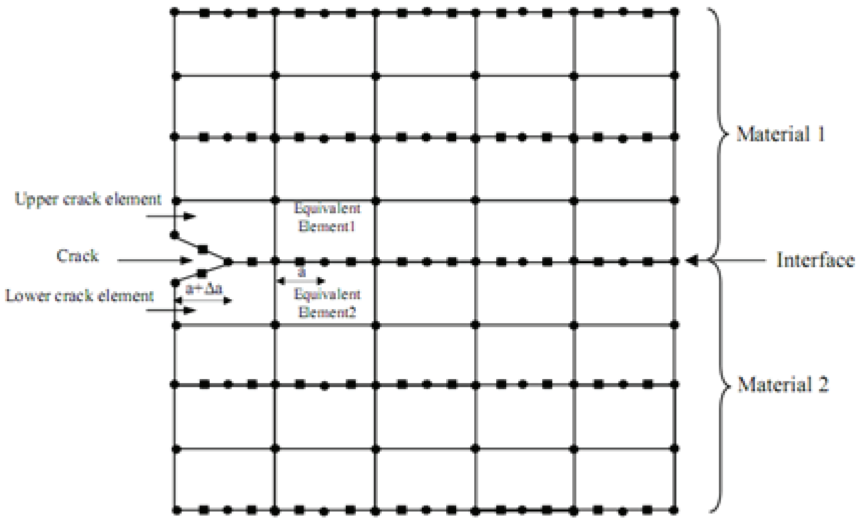

Figure 2. Mesh of cracked biomaterials

2. Numerical results and discussions

The computer program developed by incorporating the present mixed finite element (RMQ-7) is employed for the analysis of a dissimilar square plate with a center crack in the interface plan between two isotropic materials [LIN 1976] as shown in figure 3. In this problem, the present element is associated to the virtual crack extension method to evaluate the energy release rate (G). During numerical calculation, the choice of the crack length variation (Δa) is very important. To see the influence of this variation on the precision of calculation, we considered only one mesh with 50 elements and 286 degrees of freedom and we varied the extension in the interval (Δa/a=1/10÷1/500). The results obtained with present interface element are compared with the values of the analytical solution and the values of the numerical modelling of Lin and Mar [LIN 11]. These authors gave like results of their studies the stress intensity factors K_I and K_{II} from which we evaluated the energy release rate. Table 1 gives the values obtained according to E_1/E_2.

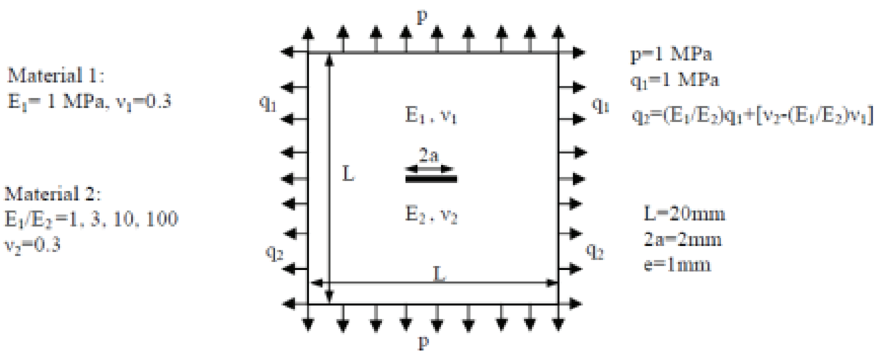

Figure 3. A center crack in a dissimilar square plate

E1/E2	Energy release rate G(N/mm)		
	Rice and Sih (1965)	Lin and Mar (1975)	Present mixed element
1	3.14	3.20	2.43
3	6.17	6.28	6.28
10	16.43	16.67	17.36
100	144.20	144.60	149.23

Table 1. Energy release rate of center interface crack between dissimilar materials

The results obtained confirm the validation of the present element for the cracked structure. The choice of the length variation of crack Δa has a very significant role on the results precision. Indeed, it is necessary that this variation is sufficiently small so that the solutions obtained u(a) and u(a+Δa) are closer as much than the Δa extension is small compared to dimensions of the crack element. To highlight the importance of the choice of the extension Δa, we made numerical tests by using several values of Δa/a. Table 2 gives the values of the energy release rate for various values of Δa/a for E1/E2=3.

	Δa/a					
	$\frac{1}{50}$	$\frac{1}{100}$	$\frac{1}{150}$	$\frac{1}{200}$	$\frac{1}{250}$	$\frac{1}{300}$
Energy release rate G (N/mm)	3,68	4,15	4,67	5,20	5,71	6.28

Table 2: Energy release rate for various values of Δa/a

The results obtained confirm the importance, which present the good choice of the crack variation length. We noted a very good stability between values (1/50) and (1/300) of the ratio (Δa/a).

3 Conclusions

A special mixed finite element is used to model cracked interfaces between two dissimilar materials. In the formulation of the present element, we used Reissner's mixed variationnel principle to build the parent element. The mixed interface finite element is obtained by successively exploiting the technique of delocalization and the static condensation procedure. This interface element was associated with the virtual crack extension method to evaluate the energy release rates using only one meshing by finite elements. The accuracy of the element is evaluated by comparing the numerical solution with an available analytical solution or numerical ones obtained from others finite elements. Results obtained from the present mixed interface element shown to be in good agreement with the analytical solutions. Comparison of the results shows the validity of this mixed finite element for the treatment of the problems of interfacial crack between two dissimilar isotropic materials.

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