The use of the strain approach to develop a new consistent triangular thin flat shell finite element with drilling rotation

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RÉSUMÉ. *Dans la présente communication, nous présentons un nouvel élément fini à facette plane. Il est le résultat de la combinaison d'un élément de membrane et d'un élément de plaque en flexion, tous deux basés sur l'approche en déformation. Nous avouons que les éléments classiques de membrane de classe C° fournissent une déflexion et une contrainte médiocre pour les problèmes où le mode de flexion est dominant. De plus, ils rencontrent des problèmes de continuité et de conformité lorsqu'ils sont connectés à des éléments de flexion (poutre et plaque) de la classe C^1. L'objectif de la présente communication est de surmonter ces problèmes lorsqu'un élément de membrane est couplé à un élément de plaque mince afin de construire un élément de coque. L'élément membranaire utilisé est un élément triangulaire à quatre nœuds, trois nœuds aux sommets du triangle et le quatrième en son centre. Chaque nœud possède trois degrés de liberté, deux translations et une rotation autour de la normale (Drilling rotation). Les coefficients liés aux degrés de liberté du nœud interne sont supprimés de la matrice de raideur élémentaire en utilisant la technique de condensation statique. Les fonctions d'interpolation des champs de déformations, de déplacements et de contraintes sont établies à partir des conditions d'équilibre. L'élément de plaque utilisé pour la construction du présent élément de coque est un élément de plaque mince triangulaire à quatre nœuds basé sur : la théorie des plaques minces de Kirchhoff, l'approche en déformation, le quatrième nœud fictif, la condensation statique et l'intégration analytique. L'élément de coque résultat de cette combinaison est robuste, compétitif et efficace.*

ABSTRACT. *In the present communication, we present a new flat shell finite element. It is the result of the combination of a membrane element and a bending element, both based on the strain based approach. We recognize that classical C° Plane membrane elements provide poor deflection and stress for problems where bending mode is dominant. In addition, they encounter a continuity and compliance problems when they are connected with a C^1 class plate elements. The objective of the present work is to surmount these problems when a membrane element is coupled with a thin plate element in order to construct a shell element. The membrane element used is a triangular element with four nodes, three nodes at the vertices of the triangle and the fourth one at its center. Each node has three degrees of freedom, two translations and one rotation around the normal (drilling rotation). The coefficients related to the degrees of freedom of the internal node are removed from the element stiffness matrix by using the static condensation technique. The interpolation functions of strain, displacements and stresses fields are established from the equilibrium conditions. Since, the plate element used for the construction of the present shell element is a triangular four-node thin plate element based on: Kirchhoff plate theory, the deformation approach, the four fictitious node, the static condensation and the analytic integration. The shell element result of this combination is robust, competitive and efficient.*

MOTS-CLÉS : *coque, plaque, modèle en déformation, condensation, drilling rotation*

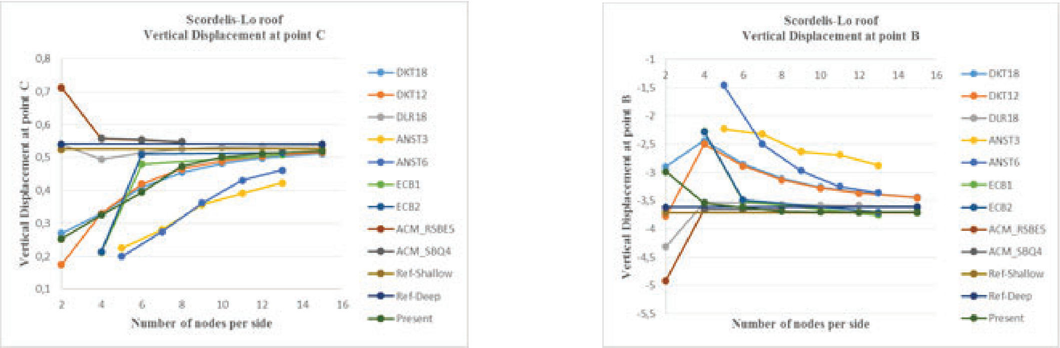
KEY WORDS: *shell, plate, strain model, condensation, drilling rotation*

1. Introduction

The present element is a flat plane thin shell element, obtained by superimposing of a membrane finite element with a thin plate finite element. The membrane element is formulated using the strain approach Himeur (2008). The interpolation functions of strain, displacement and stresses fields are developed since equilibrium conditions. Every node has three degrees of freedom: two translations and the rotation about the normal θ_z. The plate element has four nodes: three heads to which we have added a fourth imaginary node. All of nodes has three degrees of freedom. Hence the displacement fields, expressed by the use of the strain model, have twelve independent constants parameters (a_1, ..., a_{12}). The first three (a_1, a_2, a_3) are used to represent rigid body motions. The other nines (a_4, ..., a_{12}) are used to characterise the state of pure bending. They share into the deformation interpolation functions to satisfy the balances relations of kinematics compatibility for plane elasticity. Consequently, the flexure curvature fields for the higher modes derive from Pascal's triangle. For the isotropic material case, we obtain his elementary stiffness matrix by adding the stiffness matrix of the membrane element to that of the inflectional element without coupling effect. We outline the approach with his principles as follows: * we approximate the real geometry with flat planes, so we neglect the curvatures on the element. This avoids the membrane locking. * The shell element may have any orientation in the global coordinate system. * We establish the passage of the local coordinates to the global coordinates through the rotation matrix. * We rearrange the local rigidity terms (18x18) before assembly in the local level. We lift the difficulty associated with θz rigidity by introducing the rotation around the normal "drilling rotation" in the construction of the corresponding elementary stiffness matrix.

2. Results & discussions

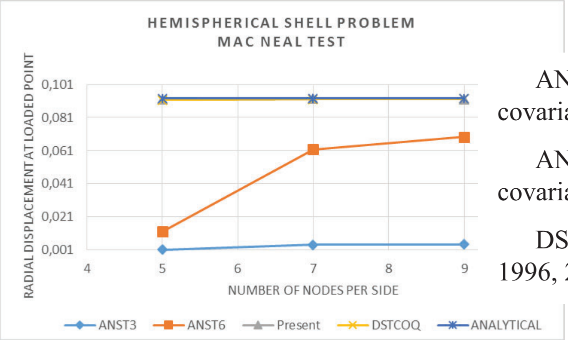
The element pass successfully both the standard Kirchhoff Patch test and the inextensional bending modes. For the usual academic tests such as: The infinitely long pinched cylinder, The Scordelis-Lo roof thin shallow shell and the square base spherical shell. The test considered here is recurrently used to investigate the performance of shell element is the Scordelis-Lo roof. It is a circular cylindrical panel where the two curved edges based on two rigid diaphragms alongside their plan and the other two edges are free. The panel is subjected to its own weight only. The results obtained by the present flat shell element for the vertical displacement at the midpoint B of the free edge, at the center C of the roof and for longitudinal displacement in A are shown at the Figures. We see a rapid convergence of the present element to the shallow shell solution compared to other elements. This remarkable convergence due to the richness of the element on membrane (cubic interpolation). The convergence curves show the good contribution of the strain based approach. The overhead results approve the good convergence of the present formulated shell element.



Vertical Displacement at point C   Vertical Displacement at point B

Cylindrical panel subjected to its weight

For the Hemispherical shell problem subjected to self-equilibrating radial point forces with 90° intervals, two inward and two outward forces at the quarter points of its open edges. We present the results for different mesh sizes. We compare our results with the analytical solution given by Flugge (1973) and the reference solution due to Belytchko (1985) and with thus the numerical given by Guenfoud (1990, 1996).They indicates that the proposed element performs well in comparison with other elements in literature.



ANST3: Three-node shell element based on assumed covariant strains (Guenfoud, 1990).

ANST6: Six-node shell element based on assumed covariant strains (Guenfoud, 1990).

DSTCOQ: Three-node flat shell element (Guenfoud, 1996, 2000).

Hemispherical shell problem (Mac Neal Test)

Radial Displacement at loaded pointe

3. Results & discussions

The formulation of a flat triangular thin shell element with a true rotation based upon the strain approach has been developed. The three translational displacement are each described in terms of cubic polynomial functions. The use of equal-order fields for all displacements has the effect of approximating further strictly the rigid body motion condition. The originality in the formulation of the present element lies in the use of the model in deformation and the use of concepts and technique in order to:

- The enrichment of the fields of displacement, thus a better precision in the approximation of the solution.
- Enhancement of behavior in case of geometric distortion of the meshes.
- The avoidance of the membrane-locking problem for curved structures.
- The response to the numerical problems induced by the absence of the rigidity related to the rotation around the normal in the case of the coplanar elements.

These concepts and techniques are:

- Adoption of the deformation approach;
- The introduction of the fourth internal node in the three-node triangular element;
- The reduction of elementary stiffness matrices using the static condensation technique;
- The use of analytical integration to evaluate the stiffness matrix.

This approach led us to a competitive, robust and efficient flat faceted shell element. The strain approach formulation demonstrate to be consistent in a very wide variety of linear analysis situations. A series of test problems conduct to evaluate the efficiency of the element compared to other elements in the literature. The results obtained confirmed the fast convergence rate of the element. The proposed element has the advantage of being simple in form and uses the six degrees of freedom. Further, it can be used for the analysis of thin shell structures, even those with complex geometries.

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